ON THE STABILITY OF A CLASS OF SWITCHED BOND GRAPHS

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ABSTRACT

This paper deals with the stability analysis of a class of switched bond graphs where all the storage components remain unchanged and in integral causality under mode switching. These properties assure the existence of a unique energy function common to all the switching modes, and the non occurrence of state jumps or discontinuities in the switched system trajectories. A result on stability of equilibria in the sense of Lyapunov is presented. The derivation of this result is done using a bond graph technique for Lyapunov stability analysis (previously developed by one of the authors of this paper) in association with results available in the literature related to the satisfaction by a candidate Lyapunov function of a sequence nonincreasing condition for the state trajectories of a general autonomous switched system. Because of its versatility, the Switched Power Junction formalism is chosen in this paper to model and simulate the commutations in the bond graph domain.

1. INTRODUCTION

Frequently in engineering problems, abrupt changes in physical systems are considered to occur instantly. This is mainly due to the facts that the behavior the engineer is interested in has a time scale much bigger than that of the abrupt change, and that the details inside the time window of this change are not relevant to the behavior under study. Thus, ignoring them results in saving time and effort. As this practice departs from the assumptions of continuity and smoothness underlying classical physics, it requires special modeling, simulation and analysis tools to handle the systems it yields, see (Mosterman and Biswas, 1998) for a detailed discussion of modeling and simulation issues related to this problem.

This paper deals with the stability analysis of switched physical systems modeled as switched bond graphs. The main tools employed to represent the commutations among the continuous bond graph (BG) models constituting the switched bond graph (SwBG) is the Switched Power Junction formalism, which has been introduced in (Umarikar and Umanand, 2005), and revisited in (Junco et al., 2007) in view of its

implementation in the simulation tool 20sim®. There are two kinds of SPJ, the $\mathbf{0_S}$ and the $\mathbf{1_S}$, which are 0- and 1-junctions admitting more than one bond *graphically* imposing effort or flow on them, respectively. The rules of causality are not violated because one and only one of these bonds is behaviorally connected (transferring power and relating variables) at a given arbitrary time instant, the others being excluded from the model. The decision on what bond to connect is made by a control variable associated to the SPJ.

A big amount of research has been dedicated to the stability of switched and, more generally, hybrid systems, as stability is both, theoretic and practically important. This problem has been approached with varied mathematical tools, ranging from differential inclusions, variational principles, multiple Lyapunov functions, Lie algebra, etc. Some relevant surveys on this subject are (Antsaklis and Nerode, 1998), (Morse et al., 1999), (Antsaklis, 2000), (Liberzon, 2003), (Margaliot, 2006).

The stability analysis of (continuous) BG has been approached in several works. An approach based on Lyapunov's Second Method and extensions (LaSalle's Invariant Principle), which exploits the system's energy, interconnection and causality features directly on the BG domain, has been presented in (Junco, 1993 and 2001), and further applied to analyze stability of equilibria and input-output passivity, as well as to synthesize stabilizing controllers for electrical machines in (Junco, 1999 and 2000) and (Junco and Donaire, 2005). More results on stability on BGs are presented in (Wu and Youcef-Toumi, 1995) and related papers by the same research group, and in (Grujic and Dauphin-Tanguy, 2000; see also previous work referenced therein). The first group of researchers presents a (non-Lyapunov) technique taylored to analyze stability properties of zero dynamics on BG. The second introduces the concept and method of energetic stability, which is Lyapunov-related but, because explicitly using state equations, it is not a pure BG-technique. For the case of linear BGs, (Morvan et al., 2003) introduced an observability-related algebraic criterion. The same work addresses a condition for the asymptotic stability of the origin being considered as a common equilibrium point (EP) of a linear, sourceless *switched BG* (i.e., an autonomous linear switched system). This result makes use of the notion of state observability from the power

variables of the R-elements of each linear BG constituting the SwBG.

In this paper we approach the stability analysis of SwBG using the energy-based methods developed by Junco for continuous BG (see (Junco, 2004) for a summary of the above cited results) immersed in a framework for general switched and hybrid system analysis as presented in, for instance, (Branicky, 1998) and (DeCarlo et al., 2000). More specifically, a class of SwBG is considered, where all the storage components remain unchanged and in integral causality under mode switching, what assures the existence of a unique energy function common to all the switching modes. Assuming that it is definite positive wrt an EP common to all the switching modes, stability in the sense of Lyapunov is proven provided that the energy function satisfies a sequence nonincreasing condition for the state trajectories.

This research is a first step towards the dynamic analysis of hybrid systems on graphical models, in our case SwBGs augmented with the graphical representation (via block diagrams or flow graphs, etc.) of the subsystems controlling or determinig their commutation rules. As shown in (Junco et al., 2007) the SPJ-formalism allows for this simultaneous representation of the complete hybrid system. Moreover, some preliminary studies, which will be the object of a future paper, indicate the feasibility of analyzing the hybrid dynamics on this representation with graphical tools in the style of (Rahmani and Dauphin-Tanguy, 2006), (Hihi and Rahmani, 2007a, b) and (Reinschke, 1988).

The remaining of this paper is organized as follows: Section 2 presents the modeling of SwBG means the SPJ formalism, addresses some previous results on stability of continuous BGs, and summarizes the background on stability of switched systems. Section 3 presents the main result of the paper concerning the stability of SwBG. Section 4 illustrates some properties and results with simulation examples and, finally, Section 5 presents the conclusions of this research.

2. BACKGROUND ON STABILITY AND SWITCHED BOND GRAPHS.

This section summarizes some background results on stability of BGs and stability of switched systems that will be used in the sequel, and presents the SPJ-formalism as the modeling tool chosen to represent the switched bond graphs.

Lyapunov Stability on Bond Graphs

To fix ideas consider the (possibly) nonlinear dynamical system $\dot{x} = f(x)$ and, without loss of generality, suppose that the state space origin is an EP, i.e., $f(0) = 0$. Analyzing the stability of the origin using Lyapunov's second method basically implies choosing a scalar *positive definite function* (pdf) $V(x)$ (written $V(x) > 0$)

and studying the sign of $L_f V(x)$ (where $L_f V(x) \equiv dV(x(t))/dt$ or, for short, $\dot{V} \equiv \dot{V}(x(t))$, is the orbital or Lie-derivative of V i.e., its time-derivative along the orbits, or trajectories, or solution paths of the state equation system). This is usually done evaluating the scalar product between the gradient of $V(x)$ and the vector field $f(x)$, i.e., $\dot{V}(x(t)) = \nabla_x^T V(x) \cdot f(x)$. If $\dot{V}(x)$ is negative semidefinite (written $\dot{V}(x) \leq 0$), then the EP is (at least) stable in the sense of Lyapunov; if $\dot{V}(x)$ is negative definite (written $\dot{V}(x) < 0$), then the EP is asymptotically stable (applies only to isolated EPs). In this latter case, $V(x) > 0$ *is* (*called*) *a Lyapunov function* for the EP. For details on Lyapunov's theory and its applications to control problems refer for instance to (Sontag, 1998).

When studying physical systems a natural candidate for a pdf $V(x)$ is the energy E stored in the system. If this assumption is satisfied, then $\dot{V} \equiv \dot{E}$ is the power flowing into the storages, and can be evaluated directly on the BG as the power flowing out of the sources minus the power into the dissipative elements. Thus, there is no need of computing the scalar product determining $\dot{V}$, nor the vector field $f(x)$, i.e., the state equations. This fact constitutes the rationale underlying the Internal Stability Analysis Procedure given as Proposition 2.1 in (Junco, 2001). Further results on the direct application on BGs of Lyapunov's Second Method are summarized next. Without loss of generality, the isolated EP considered is always the state-space origin. The results apply without restriction to BGs consisting of elements out of the basic nine-component set, but they also hold in many cases on BGs containing modulated components. **In all cases of the transcription below, the energy E is supposed to be a pdf of the states**, if not, energy-based pdfs can be considered, as formulated in (Junco, 1993).

Proposition 1 (Junco, 2004; Proposition 2.2 in [Junco, 2001[*]]).
If the energy stored in a BG is a Lyapunov function $V(x)$ (i.e., $V > 0$ and $\dot{V} < 0$), then each storage-element imposes causality on at least one R-element of the BG.

Proposition 2 (Junco, 2004; Proposition 2.3 in [Junco, 2001]).

A) The EP is asymptotically stable if the following conditions hold:
(i) all R-elements are truly dissipative, i.e., each *e-f* relationship goes through the origin and is completely contained in the first and third quadrants;
(ii) each storage in integral causality imposes causality on at least one R-element.

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[*] In the uncorrected conference version of this paper the proposition is erroneously stated as "A necessary condition for asymptotic stability". In fact it is "A necessary condition for E being a Lyapunov function".

Condition *(ii)* guarantees that the power dissipated in the R-elements depends on all the state variables; together with condition *(i)* this ensures that this dependence is a *pdf* function of the state. As $\dot{V}$ equals minus the dissipated power, the sufficient condition for asymptotic stability is satisfied.

In fact, the following stronger property holds for the simplest class of BGs:

Proposition 3 (Junco, 2004).
The EP of any connected BG (i.e., a BG not composed of disjoint BGs) constructed with elements out of the basic set and including strictly dissipative R's is asymptotically stable.
This property follows from the assumed positive definiteness of the stored energy, the strictly dissipative features of the R's, and the pure power-connected structure of this class of BGs. It can be proved applying Lyapunov's Second Method and LaSalle's Invariance Principle on BGs, see (Junco, 2001), Proposition 2.4.

Lyapunov Stability of Switched Systems

Among the many results on stability of switched systems, we will use the version of the approach via multiple Lyapunov functions presented in (Branicky, 1998), reproduced next with slight simplifications for the sake of brevity.

Theorem 2.7 (Branicky, 1998). Suppose the candidate Lyapunov functions $V_i(x)$ for the switched dynamics defined by $\dot{x}(t) = f_i(x(t))$, $i \in \{1, 2, 3, \dots, N\}$ with $f_i(0)=0$. Let **S** be the set of all switching sequences associated with the system.
If for all *i* and each switching sequence $S \in \mathbf{S}$, V_i is Lyapunov-like for f_i and the trajectory-segment in mode-*i*, and the V_i satisfy the sequence nonincreasing condition for the whole trajectory (encompassing all the modes of S), then the system is stable in the sense of Lyapunov.

Definition 2.6 (Branicky, 1998). The sequence non-increasing condition for the candidate Lyapunov functions $V_i(x)$ in Theorem 2.7 above means that

$$V_{i_{j+1}}(x(t_{j+1})) < V_{i_j}(x(t_j))$$

The ideas behind this theorem is that "these possibly multiple Lyapunov-like functions can be pieced together in some way to produce a global (nontraditional) Lyapunov function whose overall energy decreases to zero along the system state trajectories" (DeCarlo et al., 2000).

Modeling Switched BGs with the SPJ-formalism

The need to incorporate some tools to handle switching phenomena with the BG-formalism, originally conceived to handle continuous physics, was quickly recognized by the BG research community. Among the many ideas and techniques proposed to treat *idealized commutations* we have selected the Switched Power Junction or SPJ-formalism. This is because of the

advantges of the SPJs in order to circumvent some associated modeling drawbacks like varying causality of switching-modeling components, hanging junctions, failure to disconnect subsystems, and other inconsistencies. Most important for this research is the already mentioned fact that SPJs allow for a complete graphical representation of a hybrid system, via the bidirectional coupling of the SwBG with any informational graph used to explicit the subsystems determinig the commutations of the former.

Switched Power Junctions have been introduced in (Umarikar and Umanand, 2005) as a generalization or extension of standard BG-junctions. The generalized models of both $\mathbf{0_S}$ and $\mathbf{1_S}$ (the notation for 0- and 1-switched power junctions, respectively) are shown in Fig. 1. If they were standard junctions there would be a causal conflict in each of both cases. In the new formalism the causal conflict is removed via imposing the constraint that one and only one of the effort- (flow-) deciding bonds imposes the effort (flow) to the $\mathbf{0_S}$ ($\mathbf{1_S}$) at any given time instant. This convention is graphically represented by the presence of the activated bond in the junction symbol, and mathematically formalized in (1) and (2) for the $\mathbf{0_S}$ and $\mathbf{1_S}$, respectively. One and only one of the "boolean" variables in the set $\{U_1, U_2, U_3, \dots, U_N\}$ is allowed to have the value 1 at a given time instant, the rest are zero. Looking at (1) and (2) it is seen that the boolean variables assuming the zero values annihilate both effort and flow in each of the associated bonds, and thus completely disconnect all subsystems at the bond-ends opposite to the junction.
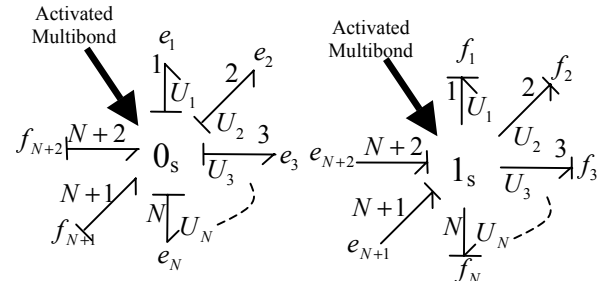

Fig. 1. Generalized $\mathbf{0_S}$- and $\mathbf{1_S}$-junctions.

Table 1. Constitutive relations of the SPJs

$\mathbf{0_S}$	$Junction\ effort = U_1 \cdot e_1 + U_2 \cdot e_2 + \cdots + U_N \cdot e_N$ $f_i = U_i \cdot (f_{N+1} + f_{N+2})$; $i = 1,\dots,N$	(1)
$\mathbf{1_S}$	$Junction\ flow = U_1 \cdot f_1 + U_2 \cdot f_2 + \cdots + U_N \cdot f_N$ $e_i = U_i \cdot (e_{N+1} + e_{N+2})$; $i = 1,\dots,N$	(2)

3. LYAPUNOV STABILITY OF SWITCHED BG

Some definitions are first introduced in order to characterize the class of SwBG this section deals with.

Definition 1. A Storage Invariant Integral SwBG (SI-I SwBG) is a switched BG where *i*) the storage elements are the same in all modes, and *ii*) all the storages are in integral causality in all modes.

Lemma 1. A SI-I SwBG has a unique energy function common to all its modes. This property follows immediately from the definition above and the fact that the energy function in a BG is fully determined by the state variables of the integral storages (the storages in integral causality).

Stability of Storage Invariant Integral SwBGs

Theorem 1. Consider a SI-I SwBG where the state space origin is an isolated EP common to all its modes. Further consider that the number of switchings in any finite time interval is finite, and that:

i) the energy function $E(p, q)$ is a (at least locally) *pdf* around the origin. Here p and q denote, respectively, the vectors collecting all generalized impulses and displacements of the SwBG.
ii) the BG_i associated to each mode is connected (i.e., it is not composed of disjoint BGs).
iii) all the R-elements are strictly dissipative.
iv) in each BG_i, each integral storage imposes causality on at least one R-element.

Then, the state space origin of the SwBG is asymptotically stable.

Proof. Because of assumption *i*) the energy function $E(p, q)$ is a candidate Lyapunov function in the sense of Theorem 2.7 of Branicky. This is a particular case where $V_i(x) \equiv E(p, q)$ is the same for all modes indexed by $i \in \{1, 2, 3, \ldots, N\}$. Assumptions *ii*)-*iv*) imply that each mode, as considered per se, is asymptotically stable (Proposition 2, Junco 2004, see above), and thus, $\dot{V}_i(x)$ decreases with time. Piecing all this together shows that the conditions for the energy function $E(p, q)$ to satisfy the sequence nonincreasing condition (Definition 2.6 of Branicky) are satisfied. □

Some of the conditions of *Theorem 1* can be relaxed and asymptotic stability still retrieved, as commented next.

Remark 1. Assumption *iv*) implies that there is at least one R-element present in each mode. This condition can be removed, imposing instead the following condition to the switching sequences:

iv-bis) in each BG_i containing R-elements, each integral storage imposes causality on at least one of them. There is no switching sequence in S such that the system remains indefinitely in a (sequence of) mode(s) without R-elements.

Indeed, the sequence nonincreasing condition is recovered if we consider as a single mode the concatenation of the mode preceding the mode without dissipation with this latter one. □

Remark 2. Assumption *ii*) can be relaxed to handle non-connected or disjoint BG_i. In this case condition *iv*) should be imposed to each sub-BG composing a disjoint BG_i. If there are sub-BGs without R-elements, condition *iv*) can again be relaxed in the sense of condition *iv-bis*). □

Remark 3. Assumption *iv*) can be relaxed if the concerned BG_i's satisfy the conditions of Proposition 3 above. □

We remark that, rigorously speaking, the class of SwBG considered here does not strictly imply a multiple, but a single Lyapunov function, and thus, we handle a particular case of Branicky's theorem. Note however that a multiplicity is associated to this single energy function through the switching dynamics. Indeed, *any switching sequence provokes* a sequence *of negative (semi)definite orbital derivatives*, i.e., a multiple orbital derivative state-function $\dot{V}_i(x) = \dot{E}(p,q)|_{Mode\ i}$.

4. A MODELING AND SIMULATION EXAMPLE

Consider the mechanical sketch of Fig. 2. Two ideal mechanical couplers indicated $Sw_{1,2}$ can be noticed, which serve to couple the mass+two-dampers system in the middle to the mass-spring-damper systems on the sides. The following switching conditions are assumed for each coupler: *i*) switch closes on contact; *ii*) switch opens when $b_{1/2}$-damper compression force becomes zero. We identify the switch binary states as follows: $Sw_i=0$, switch open (disengaged); $Sw_i=1$, switch closed (engaged). The SwBG consists then of four switching modes, corresponding to the four binary states of the pair (Sw_1, Sw_2) $\{(0, 0), (1, 0), (0, 1), (1, 1)\}$. This is represented in Fig. 3 by four separated BGs. As inactive dampers do not play any role (dampers $b_{1,2}$ when related switch in the off-state), the associated R-elements have not been retained in the corresponding BGs.
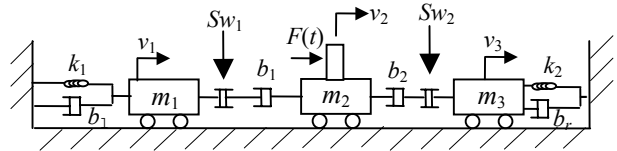

Fig. 2. Mechanical system

Sw-states	Physical system	Bond graph
(0, 0)		
(1, 0)		
(0, 1)		
(1, 1)		

Fig. 3. The four modes of the switched system

Switched Bond Graph Modeling

Fig. 4 is a SwBG representation of the example system using the (primitive) switch element or Sw-element (Strömberg, 1994, p. 86). Naturally, here all the BG-elements are present, even if they may not be active in some modes. As in the previous series of BGs, also here causality has been indicated: note that the causal stroke is pictured in the middle of a bond when this changes causality in dependence of the switch state, i.e., causality is undecided unless the switch state is specified. It means that the switched system is only specified up-to causality when using this formalism to construct the SwBG.
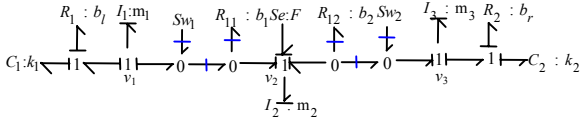

Fig. 4. The SwBG model using the ideal Sw-element.

Fig. 5 is a representation of the SwBG using the SPJ-formalism. The 0_S-SPJs are controlled by the signals u_i, which correspond to the states Sw_i as follows: $u_i = 1$ means Sw_i closed; $u_i = 0$ means Sw_i open. Consider for instance the Sw_1: when $u_1 = 1$ the 0_S ignores the effort $e_{C1}=0$ on the lower bond (associated to the auxiliary S_e with null effort), chooses the effort on the upper bond (associated to the R-element), and transmits it to the right and left bonds. Simultaneously, a net flow equal to the flow on the left bond minus the flow on the right bond is transmitted to the upper bond, and zero flow to the lower bond. This corresponds to a situation where the coupler is engaged. The opposite effort selection happens when $u_1 = 0$, meaning that zero effort is transmitted to left and right (coupler disengaged). Simultaneously, zero flow is transmitted to the R-element, what determines its inactivity during this mode. The behaviour just described is what Eq. (1) means in this particular case (see Table 1) (along with the rules on the switching conditions). Clearly, the SwBG is fully specified when using the SPJ-formalism for its representation; see (Junco et al., 2007) for other examples, including changing causality in storages and R-elements.
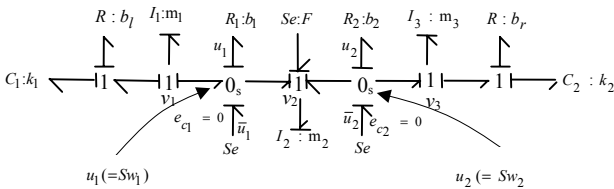

Fig. 5. SPJ- representation of the SI-I SwBG model associated to the example of Fig.2.

Stability Analysis of Switched Bond Graph

As the storages remain invariant and in integral causality under switching (only the two R-elements with coefficients $b_{1,2}$ become inactive here), the SPJ-BG of Fig.5 is a SI-I SwBG. The state vector is the same in all modes $x^T = [q_1, p_1, p_2, p_3, q_2]$, and so is the energy function. Clearly, all the BG_i share the state space origin as an isolated EP, i.e., it is a EP common to all the modes. Observe that the position of mass m_2 is not a BG state, if it were the EP would not be isolated. Naturally, the inertias are linear, but even if so depicted in the figure, the springs do not need to be linear. We do not write them down, but simply assume possibly nonlinear constitutive laws of the C's such that the energy function is a pdf of the states, so that assumption (i) of Theorem 1 is satisfied. Considering pure dissipative (also not necessarily linear) R-elements, assumption (iii) is also satisfied. Assumption (ii) is not satisfied because $BG_{(0,0)}$ (the BG of mode (0,0)) is disjoint. Assumption (iv) is not satisfied because (a), none of the two C's imposes causality in a R-element, and (b), in mode $BG_{(0,0)}$ the mass m_2 is isolated without R. The first situation is solved by Remark 3 and, regarding the second, we are in the situation foreseen in *Remark 2* with a condition similar to *iv-bis*, because, when out of equilibrium, mass m_2 will eventually contact either damper b_1 or b_2 and this mode will be abandoned in finite time.

All the above means that, by virtue of Theorem 1 and the modifications foreseen in Remarks 1-3, the state space origin of the switched system given in Fig. 2 is an asymptotically stable equilibrium point.

We calculate next the orbital derivatives of the energy function in the different modes. The energy function itself is:

$$E(p,q) = E_1(q_1)+E_2(q_2)+ p_1^2 / 2m_1 + p_2^2 / 2m_2 + p_3^2 / 2m_3$$

To get a simpler formula for the orbital derivative it is better to perform an obvious change of state variables and consequently write the energy function as:

$$V(v,q)=E_1(q_1) + E_2(q_2) + m_1v_1^2 / 2 + m_2v_2^2 / 2 + m_3v_3^2 / 2$$

Clearly, with the assumptions made on the constitutive laws of the C-elements, the energy function is a pdf of the states. Its orbital derivative in each of the modes is as follows:

$$\dot{V}_{(0,0)}(v,q) = - b_l v_1^2 - b_r v_3^2$$

$$\dot{V}_{(1,0)}(v,q) = - b_l v_1^2 - b_r v_3^2 - b_1(v_1 - v_2)^2$$

$$\dot{V}_{(0,1)}(v,q) = - b_l v_1^2 - b_r v_3^2 - b_2(v_2 - v_3)^2$$

$$\dot{V}_{(1,1)}(v,q) = - b_l v_1^2 - b_r v_3^2 - b_1(v_1 - v_2)^2 - b_2(v_2 - v_3)^2$$

It is clearly seen that *a multiple orbital derivative is obtained*. It is a sequence of negative semidefinite functions of the state, as none of them depend on the q-variables in any of the modes, because none of the C-elements imposes causality to any of the R-elements in any of the BG_i.

Simulation Results

The SPJ-model of Fig. 5 has been implemented in the 20sim™ simulator, and a simulation experiment has been conducted with the example system parameterized

as follows: $b_1 = 1$; $b_2 = 1$; $b_l = 0.1$; $b_r = 0.5$; $k_1 = 0.1$; $k_2 = 2$; $m_1 = 1$; $m_2 = 4$ and $m_3 = 1$. The initial conditions of the experiment are: $x_1(0) = 0.2$; $x_2(0) = 0.8$; $x_3(0) = 1.1$; where x_i is the position of the i-th mass. The left end of damper b_1 is at a distance $z_1(0) = 0.15$ to the left of mass m_2; the right end of damper b_2 is at a distance $z_2(0) = 0.05$ to the right of mass m_2; $v_1(0) = -0.025$. SI-units are used for all magnitudes –but are not given here.

Figs. 6 and 7 plot the positions and the speeds of the three masses. With masses 1 and 3 at rest and mass 2 moving (at constant speed $v_1 = -0.025$) to the left, the system is initially in mode (0,0). In this mode the energy function remains constant and no power is dissipated, cf. the expression of $\dot{V}_{(0,0)}$ above (where v_3 and v_1 are zero), as well as Fig. 8 below. This latter figure also shows that the switches are open ($U_1 = Sw_1 = 0$, $U_2 = Sw_2 = 0$). The system commutes to mode (1,0) when the left end of damper b_1 gets in contact with mass 1, which initiates an oscillatory movement, while mass 2 decelerates first and inverts its motion later. Most of the energy originally stored in the system gets dissipated (in dampers b_1 and b_l) during this mode, cf. $\dot{V}_{(1,0)}$ (where only v_3 is zero) and Fig. 8, which also shows that the control signal of switch 1 is high, i.e., $U_1 = Sw_1 = 1$.
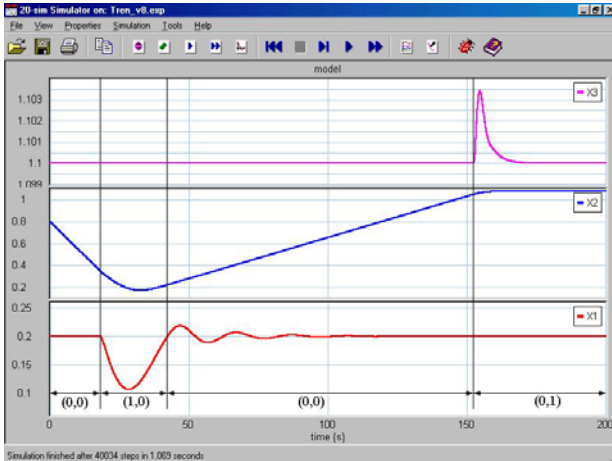

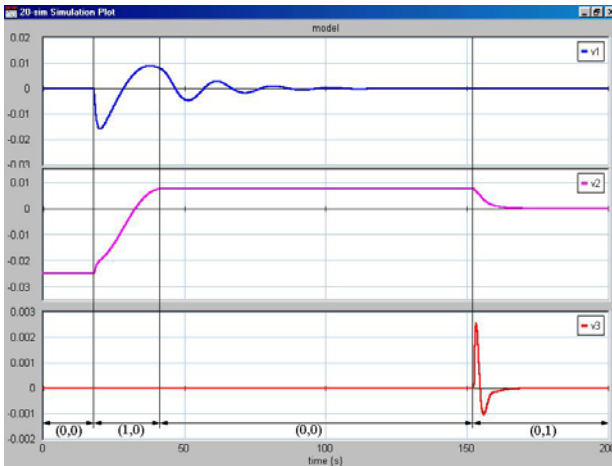**Fig. 6.** Time trajectories of the mass positions.

Fig. 7. Time trajectories of the mass velocities.

The back oscillation of mass 1 pushes mass 2 to the right and brings the switch 1 to its disconnection condition, so that the system commutes back to mode (0,0), but with a different energy configuration: while mass 2 moves at (positive) constant speed, the left mass-spring-damper subsystem is (slightly) excited, what means that (a small amount of) energy continues to get dissipated, cf. $\dot{V}_{(0,0)}$ (v_3 is zero, but not v_1). After a while, the right end of damper b_2 contacts mass 3, the switch 2 goes in the on-state (see U_2 in Fig. 8), and the system commutes to mode (0,1). With the movement of the left subsystem already faded ($v_1 = 0$), energy is only dissipated in the coupled subsystem on the right, cf. $\dot{V}_{(0,1)}$, where only $v_1 = 0$, and Fig. 8.
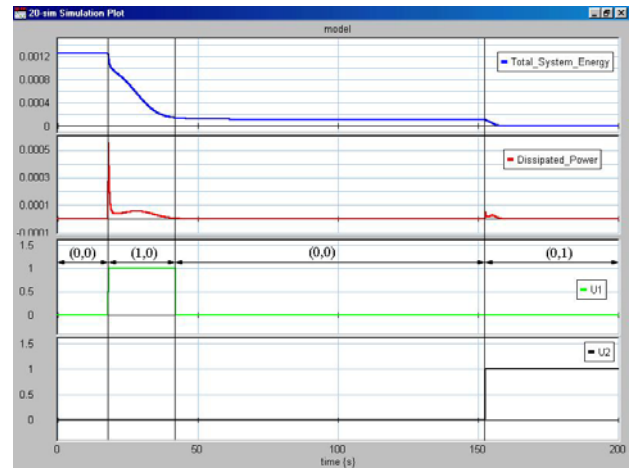

Fig. 8. The energy function, the power dissipated in the system, and the control signals of the switches ($U_i = Sw_i$)

The first plot of Fig. 8 shows the nonincreasing nature of the energy, which is the candidate Lyapunov function in this problem. It converges to zero, what means that the state converges to the origin, what in turn proves the asymptotic stability of this EP. This convergence of the state variables can be partially seen on the speeds (Fig. 7). The complementary conditions, namely that the deformations of the springs also go to zero, are also verified, even if not shown on the plots.

5. CONCLUSIONS

The stability of a particular class of switched bond graphs has been analyzed with Lyapunov-like tools applied directly in the BG-domain. The study is restricted to BG where the storages remain invariant under switching, both in their constitution and causality. Further research will focus on switched bond graphs admitting derivative causality when switching among modes. This implies the lost of a unique energy function common to all modes of the BG, the possible variation of the system order when switching, and the appearance of discontinuities in the state trajectories.

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