ON NUMERICAL ESTIMATION OF PERFORMANCE ROBUSTNESS OF TRANSPORTATION NETWORKS

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ABSTRACT

The limited economic and spatial resources require a careful planning of investments, so that achieving the most accurate knowledge of the performances of any proposed solution is a problem of worth importance. A possible solution to this problem is provided by *sensitivity analysis* to evaluate their *robustness* to uncertainties. Then, in this paper, traffic simulations will be used to evaluate, by means of a proposed sensitivity analysis approach, the robustness of several interventions planned for the real case study consisting of the transportation network around the seaport of Genoa in Italy.

INTRODUCTION

In the last century, the continuous increase in traffic demand has been faced by firstly building new infrastructures and then, especially in the last two decades, by optimizing the performances of the already existing ones.

Therefore, it is well known that the limited economic and, especially in urban areas, spatial resources require a careful planning of investments, so that achieving the most accurate as possible knowledge of the performances of any proposed solution is a challenge to win. In this sense, the problem becomes more difficult whenever not only the present transport demand, but also the "estimated" future one, have to be taken into account.

Then, in this framework, most approaches for improving transportation network performances share the need of knowing, and often forecasting the traffic flows in the different links of the network itself. Despite this need, the problem of estimating traffic demand is difficult to be solved, especially when large and complex networks are taken into account, since traffic demand is affected by many sources of uncertainty. However, there are several different approaches for data acquisition, mainly characterized by different *temporal* and/or *spatial* levels of aggregation. In detail, from the temporal point of view, traffic data may be:

- *off-line estimated at once*, and eventually refined with periodical data sampling;
- *continuously acquired* by means of sensors which count and/or classify vehicles.

On the other hand, from the spatial point of view, traffic data may be acquired

- *globally*, i.e., by means of carefully planned measures in the whole considered network, thus leading to an "expensive" Origin-Destination (OD) matrix definition;
- *locally*, i.e., considering only the vehicle flows in particular locations and trying to estimate the OD-matrix by means of (relatively) few samples.

Furthermore, data may be aggregated with respect to the kinds of users, transportation modes, and so on.

Therefore, while the kinds of data to be collected depend on the particular problem to be faced, normally it is not usually possible to achieve a perfect knowledge of the origin-destination demand, as shown in recently published paper concerning Origin/Destination (O/D) matrix estimation, such as in Lundgren and Peterson (2008) and in Nie and Zhang (2008).

In this framework, finding a procedure which helps to choose, among the different possible improvements and modifications, and under uncertain mobility demand, the best interventions that may be implemented within budget and spatial constraints, is an interesting problem to solve.

A possible solution to this problem is provided by *sensitivity analysis* (see, for instance, Saltelli et al. (2004) for a general view of the problem), which, for what concerns transportation problems, dates back to the eighties (see, for instance, Tobin and Friesz (1988)).Then, although many of the achieved results to be formidable analytic methodologies, it is often difficult to apply them to real cases. In effect, the performances of transportation networks depend on structural/technological elements and on their interactions with the traffic demand in a way frequently difficult to be analytically described, especially when large networks are considered. This is the reason why, in practice, simulation is typically used in designing transportation infrastructures and in planning transportation services, instead of analytical modelling.

Then, in this paper, traffic microsimulation will be used for providing traffic data for sensitivity analysis of

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a real case study consisting of the transportation network around the seaport of Genoa, in Italy. Note that, being the basic ideas concerning the considered sensitivity analysis approach already presented in Di Febbraro and Sacco (2008), the aim of the present paper is to test, by means of numerical computations, the goodness of the methodology.

SENSITIVITY ANALYSIS

In the following sections, after giving a suitable definition of urban transportation network, the basics of the considered sensitivity indices are introduced.

Main definitions

The definition of *Urban Transportation Network (UTN)* here given has the aim of pointing out the interaction between the capacity of the infrastructures and the mobility demand. Then, for completeness of the description, the proposed approach, in the given definition the civil structures (roads, intersections, etc.), and the technological devices (such as traffic light controllers, Variable Message Signs (VMS), etc.) are considered separately.

Definition 1 Urban Transport Network
An UTN is the 4-uple

$$UTN = \{\mathcal{C}, \mathcal{I}, \mathcal{K}, \mathcal{M_{OD}}\} \tag{1}$$

where $\mathcal{C} = \{C_j, j = 1, \ldots, J_C\}$ *is the set of the centroids representing the different zones of the considered area, whereas* $\mathcal{I} = \{I_j, j = 1, \ldots, J_I\}$ *and* $\mathcal{K} = \{K_j, j = 1, \ldots, J_K\}$ *are the set of the civil infrastructures and of technological devices "working" in the considered area, respectively. Finally, the term* $\mathcal{M_{OD}} \in \mathbb{R}^{J_C, J_C}$ *is the Origin/Destination (O/D) matrix, whose generic element* $\mathcal{M_{OD}}(i, j)$ *is a non-negative real number representing the mobility demand from the centroid* i *towards the centroid* j*. In this definition, the elements representing the mobility demand with origin and destination in the same centroid are neglected, that is* $\mathcal{M_{OD}}(j, j) = 0, \forall\, j = 1, \ldots, J_C$. ■

Note that the above definition gathers both the *structural* and *technological* elements of a general urban traffic network, together with the relevant mobility demand.

Then, consider a set of m different network improvements which may be applied to an UTN. As said, these changes may be adopted in both the elements defining the transportation supply or network capacity, and may consist, for instance, of the introduction of new traffic controllers, or of the modification of the viability and the mobility demand, perhaps due to the modal shift towards public or railway transportation. Afterwards, given m different possible improvements, then there are $M_c = 2^m$ possible configurations, in which one or more of the improvements are implemented at a time. Then, it is possible to define the *vector of all the possible configurations* as follows.

Definition 2 Configuration Vector
The vector of all the possible configuration is defined as

$$s = \left[\begin{array}{cccc} s_1 & s_2 & \ldots & s_{M_c} \end{array}\right] \tag{2}$$

in which each element is a sequence

$$s_i = \{s_i(k),\, k = 1, \ldots, m\}, \tag{3}$$

whose k^{th} *element* $s_i(k)$ *is set to 1 (that is,* $s_i(k) = 1$*) or to 0 (that is,* $s_i(k) = 0$*), whether the* k^{th} *possible intervention is considered or not in the* i^{th} *configuration, respectively.* ■

From the above definition, it results that the sequences

$$s_1 = \overbrace{\{0\ \ldots\ 0\}}^{m}, \quad \text{and} \quad s_{M_c} = \overbrace{\{1\ \ldots\ 1\}}^{m}, \tag{4}$$

represent the present network without any intervention and the configuration characterized by the presence of all the interventions, respectively.

For what concerns the O/D matrix, it is useful to define an *Origin-Destination vector (OD-vector)* as follows.

Definition 3 Origin-Destination vector
Let J_C *be the number of centroids in the considered UTN, and assume that the couples with the same origin and destination centroid, that is those corresponding to the diagonal elements* $\mathcal{M_{OD}}(j, j) = 0$*, are not considered. Then, the OD-vector is defined as*

$$OD = \left[\begin{array}{cccc} od_1 & od_2 & \ldots & od_{n_{OD}} \end{array}\right], \tag{5}$$

gathering the $n_{OD} = J_C(J_C - 1)$ *demand elements* od_j*,* $j = 1, \ldots, J_C$*, corresponding to the non null elements of* $\mathcal{M_{OD}}$. ■

Note that while some changes in an UTN may lead to the rising or the suppression of centroids, in this paper, for the sake of simplicity, the number of centroids J_C is considered to be constant. Nevertheless, such an assumption does not imply that mobility demand is also constant, since the elements of the OD vector can vary.

Performance index definition

Consider now the problem of evaluating the performances of an UTN for all the possible configurations in s. To this end, it is possible to define the following *performance index*, which explicitly depends both on the configurations and the mobility demand.

Definition 4 Performance vector
Let $\phi : s \times OD \to \mathbb{R}$ *be a generic traffic performance function. Then, the performance vector is defined as*

$$\Phi(s, OD) = \left[\begin{array}{ccc} \phi(s_1, OD) & \ldots & \phi(s_{M_c}, OD) \end{array}\right], \tag{6}$$

which gathers all the values assumed by the performance parameter $\phi(s_i, OD)$*, which is computed for all the possible configurations, and by assuming the demand gathered in the vector* OD*. By this definition, the performance vector* $\Phi(s, OD)$ *results to have the same number of elements of the configuration vector* s. ■

Once computed the above vector, it is easy to determine the best UTN configuration i^*, since, depending of the chosen performance index, it is the one corresponding to the minimum or to the maximum element of the vector (6), that is

$$i^* = \arg\left[\min_{i=1,\ldots,M_c} \phi(s_i, OD)\right], \tag{7}$$

or

$$i^* = \arg\left[\max_{i=1,\ldots,M_c} \phi(s_i, OD)\right], \tag{8}$$

respectively.

For instance, if the performance index ϕ gives the mean travel time in the UTN, then the best configuration is given by (7). On the other hand, if ϕ represents the mean vehicle speed, then the best configuration is given by (8).

Note that the performances parameter ϕ may be a suitable combination of different indices, thus taking into account, at a time, various aspects of the traffic problem, such as the mean travel time, the delay time, some pollution indices, the intervention building cost, a suitably defined cost-benefit ratio, and so on.

Definition of Sensitivity

Let consider the robustness of the traffic performances. As said, the elements of the vector OD are affected by uncertainties so that the *confidence* of the results in the vector $\Phi(s, OD)$ have to be carefully evaluated.

In order to cope with this problem, each element $\phi(s_i, OD)$ of the performance vector $\Phi(s, OD)$ may be associated to a set of *sensitivity measures* evaluated with respect of the demand od_j, $j = 1, \ldots, J_C$, uncertainty. Such indices are defined as the ratios

$$S_\Phi(i,j) = \frac{\dfrac{\partial\phi(s_i, OD)}{\phi(s_i, OD)}}{\dfrac{\partial od_j}{od_j}}, \tag{9}$$

$i = 1, \ldots, M_c$, $j = 1, \ldots, n_{OD}$, which express, for each configuration s_i, the relative deviation of the performance index $\phi(s_i, OD)$ computed with respect to its nominal value, compared with the relative variation of the term od_j.

Note that, in order to compute such values, Eq. (9) may be written as

$$S_\Phi(i,j) = \frac{\partial\phi(s_i, OD)}{\partial od_j} \cdot \frac{od_j}{\phi(s_i, OD)}, \tag{10}$$

$i = 1, \ldots, M_c$, $j = 1, \ldots, n_{OD}$, where:

- the partial derivative expresses how *amplified* or *reduced* is the variation of the performances when the demand od_j varies;
- the term od_j is the *nominal demand* for which the network improvements are designed;
- the term $\phi(s_i, OD)$ is the *nominal performance*, that is, the network performances computed at the nominal demand with the configuration s_i.

Then, the above sensitivity indices may be gathered in the following compact representation.

Definition 5 Sensitivity Matrix
The matrix $S_\Phi \in \mathbb{R}^{M_c, n_{OD}}$, defined as

$$S_\Phi = \begin{bmatrix} S_\Phi(1,1) & \cdots & S_\Phi(1, n_{OD}) \\ \vdots & \ddots & \vdots \\ S_\Phi(M_c, 1) & \cdots & S_\Phi(M_c, n_{OD}) \end{bmatrix} \tag{11}$$

and gathering all the sensitivity parameters, is said "sensitivity matrix". ■

Then, once computed the sensitivity matrix S_Φ, it is possible to choose the most robust configuration with respect to the uncertainty of OD. In fact, it corresponds to the row which fulfills the minimum matrix norm criterion, that is

$$i^* = \arg\left[\min_{i=1,\ldots,M_c} ||S_\Phi(i,:)||_p\right], \quad p = 1, 2, \infty, \tag{12}$$

where $S_\Phi(i,:)$ is "Matlab-like" notation indicating the i^{th} row of the sensitivity matrix S_Φ, while the subscript p indicates the chosen norm.

For what concerns the meaning of the different norms in (12), it is worth saying that

1. when $p = \infty$, the norm coincides with the greatest element of the argument vector, that is

$$||S_\Phi(i,:)||_\infty = \max_{j=1,\ldots,n_{OD}} |S_\Phi(i,j)| \tag{13}$$

   Such a norm may be used to put into evidence the most critical situation, but does not takes into account the overall performance of the net;

2. when $p = 2$, the norm coincides with the Euclidean norm

$$||S_\Phi(i,:)||_2 = \sqrt{\sum_{j=1}^{n_{OD}} [S_\Phi(i,j)]^2}.$$

   In this case, due to the squared values, the greatest elements of the row i weight more than the smaller ones. In particular, the contribution of the elements in the interval $(-1, 1)$ is reduced, while those in $(-\infty, -1)$ and $(1, +\infty)$ is amplified;

3. when $p = 1$, the norm is given by the relation

$$||S_\Phi(i,:)||_1 = \sum_{j=1}^{n_{OD}} |S_\Phi(i,j)|.$$

   In this case, all the elements of the vector $S_\Phi(i,:)$ are taken into account without any amplification or attenuation. This norm is useful when distortions in the sensitivity matrix have to be avoided.

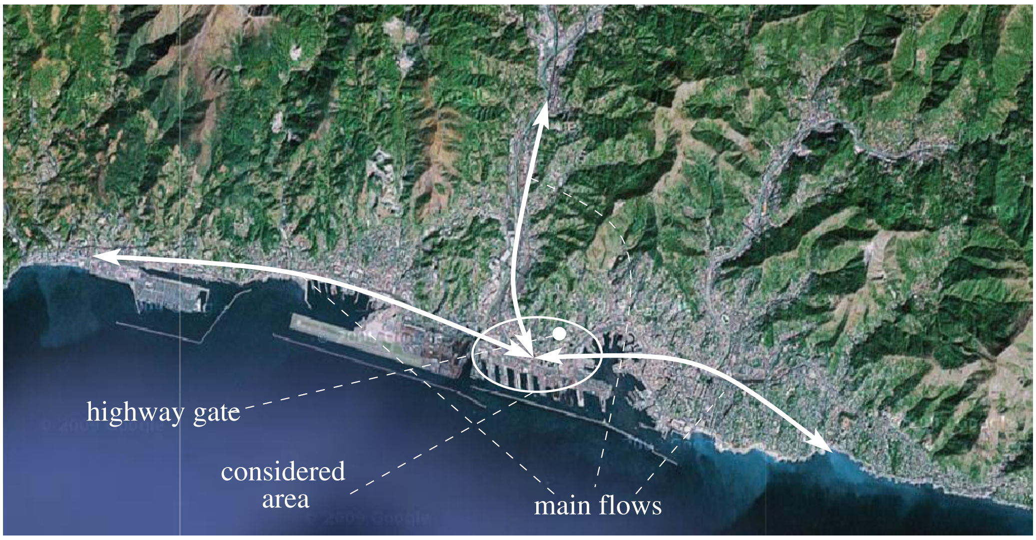

Figure 1: View of Genoa.

Although the above defined index $\Phi(s, OD)$ and the relevant sensitivity matrix S_Φ give useful information about the best configuration to choose, they cannot be often easily computed due to the lack of analytical traffic models capable to take into account all the UTN aspects. In order to tackle with this problem, in the following section a simple procedure to compute the elements of the sensitivity matrix by means of a simulation approach is described.

Computing the sensitivity elements

In order to compute the sensitivity values of Eq. (11), the derivative in Eq. (9) may be approximated with the incremental ratio

$$\frac{\partial\phi(s_i, OD)}{\partial od_j} \simeq \frac{\phi(s_i, OD + \Delta_j) - \phi(s_i, OD)}{\delta}, \quad (14)$$

where

$$\Delta_j = \begin{bmatrix} \overset{1^{st}}{\downarrow} & \overset{\dots}{\downarrow} & \overset{j^{th}}{\downarrow} & \overset{\dots}{\downarrow} & \overset{n_{OD}^{th}}{\downarrow} \\ 0 & \dots & \delta & \dots & 0 \end{bmatrix}, \quad (15)$$

is a vector in which all the elements are null, but the j^{th} one, that assumes a small value δ.

With this approximation, it is possible to estimate the values of ϕ in (s_i, OD) and $(s_i, OD + \Delta_j)$, $i = 1, \dots, M_c$, $j = 1, \dots, n_{OD}$, by means of simulation. Then, the number of operations necessary to compute all the elements of the sensitivity matrix is

$$\#_{op} = M_c(n_{OD} + 1)\nu, \quad (16)$$

where ν is the number of statistical independent simulations which guarantees a good confidence of each parameter ϕ (see, for instance Law and Kelton (1991), for further details).

CASE STUDY

In this section, the formerly proposed methodology is applied to a real case study consisting of a portion of the urban transportation network of the city of Genoa (North-West of Italy) around the seaport.

The Genoa seaport transportation network

The considered case study consists of the urban area pointed out in Fig.1, which essentially represents the "junction" among the three high populated areas into which Genoa is divided.

In effect, as depicted in Fig.1, it consists of the portion of the Genoan urban network shared, at a time, by the freight traffic flows traveling from (resp., towards) the seaport towards (resp., from) the highway gate and the private/public traffic flows traveling from (resp., towards) the east side of the city towards (resp., from) the west one.

The choice of such a case study is motivated by the following reasons:

1. the considered network is heavily influenced by the demand of mobility of goods by means of trucks, which use part of infrastructure to reach the highway from the seaport gates and vice-versa;

2. as said, the considered area is also interested by the mean flows coming from the west and the north sides of the city and aiming to reach the center or the east side, thus resulting heavily congested in the rush hours;

3. some interventions are planned for this area. They mainly consist of improving the capacity of roads and roundabouts.

Then, to the aim of evaluating the performances of the planned interventions, and the relevant robustness, the O/D matrices of private users (cars and motorcycles)

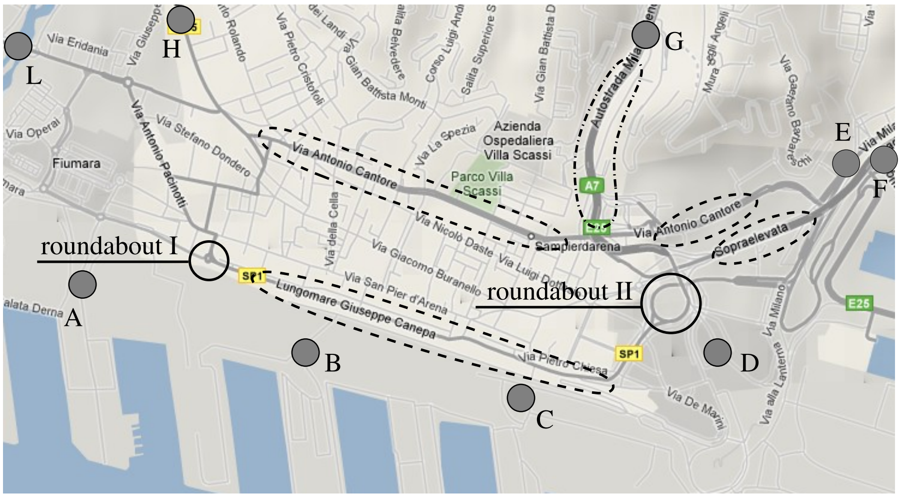

Figure 2: View of the transportation network around the Genoa seaport.

have been *estimated* by means of several measures, repeated for some days in different places of the network, and a suitable traffic simulation software. On the contrary, the freight O/D matrix has been estimated by means of interviews to the seaport operators. As a consequence, the O/D matrix of the private users resulted to have low uncertainty (less than 5%, since measures may be accurately taken and sufficiently repeated), while the freight demand is somewhat more uncertain.

Present configuration
Consider the present configuration of the chosen network depicted in Fig. 2. In such a figure, the main elements are:

- centroids representing seaport gates (A, B, C, D);
- centroids representing the origin/destination of the private/public vehicles flows traveling in the area (E, F, H, L);
- a centroid representing the highway gate (G);
- the main, and critical from the point of view of their capacity, roundabouts (roundabout I and roundabout II);
- the main roads, indicated by means of dashed lines;
- the highway ramp, indicated by means of a dashed/dotted line.

Alternative Configurations
For what concerns the interventions which may be implemented in the considered area, they consist in making the road widths uniform, where possible (this intervention is already planned and essentially consists of creating the second lane in the road nearest to the seaport gates), and in the two following interventions:

1. the redesign of roundabout I, aiming at avoiding the left turn of vehicles coming from west. After this intervention, such vehicles, which represent a relatively negligible portion (less than 10%) of the whole flow, have to reach their destinations by means of alternative paths;
2. the change of the “right to way” in roundabout II aiming at limiting the flows conflicts, and thus reducing the probability of traffic jams. Also in this case, some users have to choose alternative paths to reach their destinations.

Therefore, there are four admissible configurations for the considered case study:

s_0: the present configuration, without interventions;

s_1: in this configuration only roundabout I is modified;

s_2: in this configuration only roundabout II is modified;

s_3: in this configuration both roundabouts are modified;

Note that in configuration s_i, $i = 1, 2, 3$, also the road enlargement is considered to be implemented, whereas it is not considered in configuration s_0.

Then, the problem is comparing the different configurations and evaluating whether the increased travel times of users are balanced by a significant benefit for the whole network.

Simulation model

Then, for what concerns the simulation model used to compute the performance index and the sensitivity index, it consists of the TSS Aimsun model reported in Fig. 3, where, for the sake of clearness, the map layer is not depicted.

Numerical results
In this section, some numerical results relevant to the rush hour between 8.00 a.m, and 9.00 a.m. are discussed. In doing so, since in the chosen area the main problem is due to the trucks traffic entering/exiting the seaport gates,

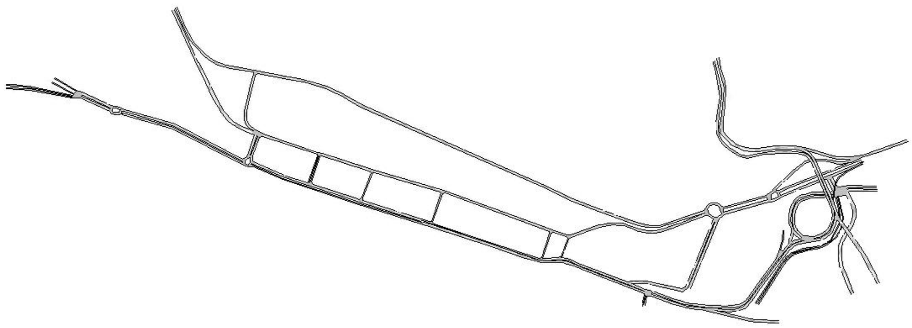

Figure 3: AIMSUN model of the transportation network in the considered area.

in the discussed analysis only the effects of the uncertainty of such a demand are considered. In doing so, the demand vector OD resulted to have only 20 elements, corresponding to the 20 following non zero elements of the truck origin-destination matrix:

$od_{A,B}$=2.0	$od_{A,D}$=3.7	$od_{A,L}$=10.7
$od_{C,A}$=9.3	$od_{C,B}$=1.3	$od_{C,D}$=1.7
$od_{C,F}$=1.0	$od_{C,L}$=5.0	$od_{D,G}$=3.0
$od_{D,L}$=40	$od_{F,L}$=20.7	$od_{G,A}$=2.0
$od_{G,C}$=3.0	$od_{G,D}$=1.3	$od_{G,L}$=16.7
$od_{L,A}$=10.0	$od_{L,C}$=12.0	$od_{L,D}$=22.7
$od_{L,G}$=42.3	$od_{L,H}$=5.3	-

Then, for what concerns the simulation results, they have been obtained by means of ν repeated simulation runs, thus reducing the effects of the stochastic variables of the model and helping to obtain a good estimate of real performance values $\phi(s_i, OD)$, $i = 1, \dots, M_c$, hereafter indicated with the general compact notation θ_i.

Obviously, as said, simulations only allow to compute an estimate of the real parameters θ_i, just providing the sample mean

$$\hat{\theta}_i = \frac{1}{\nu}\sum_{h=1}^{\nu} \theta(h)_i, \quad i = 1, \dots, M_c, \tag{17}$$

where $\theta(h)_i$ is the value of the considered parameter computed, for the configuration i, during the h^{th} simulation run.

Finally, to compare the different configurations, a measure of the "distance" of the real parameter θ_i from its estimate $\hat{\theta}_i$ has been obtained by computing the confidence interval I, which is defined as the interval including the real parameter θ_i with an assign probability $(1 - \alpha)$, that is

$$\mathbb{P}\{\theta_i \in I\} \geq 1 - \alpha. \tag{18}$$

With this formulation, the confidence interval is

$$I = [\hat{\theta}_i - t_{\alpha/2,\nu-1}\hat{\sigma}_{\theta_i}, \, \hat{\theta}_i + t_{\alpha/2,\nu-1}\hat{\sigma}_{\theta_i}], \tag{19}$$

where $\hat{\sigma}_{\theta_i}$ is the sample standard deviation

$$\hat{\sigma}_{\theta_i} = \frac{1}{\nu(\nu-1)}\sqrt{\sum_{h=1}^{\nu}(\theta(h)_i - \hat{\theta})^2}, \tag{20}$$

$i = 1, \dots, M_c$, and $t_{\alpha/2,\nu-1}$ is the parameter of the t-Student distribution with confidence α and $\nu - 1$ degrees of freedom. In this case, chosen $\alpha = 0.05$, satisfactory results have been obtained with $\nu = 6$.

Then, in Tab. 1 the sample mean and the confidence interval, computed as above recalled, of some important traffic performance parameters are reported. Looking at such results, it is easy to note that configuration s_1 is unstable, being its sample standard deviation (with these values it is unrealistic to compute the confidence interval) very high. In effects, the simulations of such a configuration are characterized by frequent jams, as, however, confirmed by the real network behavior. For what concern the best configuration, s_1 looks to be the best one, being characterized by the best value of 4/5 of the chosen performance parameters. Nevertheless, looking at the confidence intervals in Tab. 1, it is possible to note that all the proposed solutions lead to almost the same benefit with respect to the present configuration.

On the other hand, the norms of the sensitivity matrix indicates that the more "robust" configuration is s_3 for 4/5 of the parameters. Such a result suggests to implement such a configuration, which guarantees the stablest improvement of the network performances, with respect to the freight traffic uncertainty.

Conclusions

In this paper, some numerical results relevant to the sensitivity analysis of different configurations of interventions on the traffic network around the seaport of Genoa has been presented. In doing so, the proposed sensitivity indices has been computed by means of traffic simulation. The obtained numerical results show that, although all the configurations of interventions give almost the same benefit to the whole network, one of them results to be more robust with respect to the uncertainty of freight mobility demand in the considered area. To conclude, such results point out the importance of having a tool, such as the one here proposed, for robustness analysis in transportation.

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Table 1: Mean values and standard deviation of the configuration performances.

	unstable configuration		stable configurations					
	s_0		s_1		s_2		s_3	
	Mean	Std. Dev.	Mean	h.l. (95%)	Mean	h.l. (95%)	Mean	h.l. (95%)
Delay Time (s/km)	302.16	239.52	**39.49**	1.64	40.68	2.75	40.88	2.17
Density (veh/km)	55.58	45.18	**5.87**	0.17	6.16	0.13	6.1	0.43
Flow (veh/h)	1497.2	1136.46	2768.6	57.57	**2775**	58.63	2752.8	164.43
Speed (km/h)	22.67	11.14	**35.26**	0.57	34.99	0.34	35.08	0.31
Stop Time (s/km)	292.89	245.47	**23.89**	2.2	24.97	2.35	25.46	2.25

Table 2: Norms of the sensitivity matrix.

	$\|\|S_\Phi\|\|_1$			
	s_0	s_1	s_2	s_3
Delay Time (s/km)	13.86	2.4	2.38	**2.02**
Density (veh/km)	1.39	1.14	2.82	**0.9**
Flow (veh/h)	38.22	0.93	1.23	**0.72**
Speed (km/h)	**0.3**	0.45	0.57	0.37
Stop Time (s/km)	26.14	3.93	4.54	**3.04**

	$\|\|S_\Phi\|\|_2$			
	s_0	s_1	s_2	s_3
Delay Time (s/km)	4.62	0.7	0.73	**0.59**
Density (veh/km)	0.41	0.35	1.66	**0.27**
Flow (veh/h)	11.98	0.29	0.34	**0.21**
Speed (km/h)	**0.08**	0.15	0.16	0.11
Stop Time (s/km)	8.61	1.11	1.24	**0.89**

	$\|\|S_\Phi\|\|_\infty$			
	s_0	s_1	s_2	s_3
Delay Time (s/km)	2.72	0.42	0.37	**0.3**
Density (veh/km)	0.19	0.21	1.62	**0.17**
Flow (veh/h)	7.64	0.18	0.13	**0.11**
Speed (km/h)	**0.05**	0.08	0.07	0.05
Stop Time (s/km)	5.46	0.63	0.56	**0.43**

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