MODE-INDEPENDENT STATE FEEDBACK DESIGN FOR DISCRETE SWITCHED LINEAR SYSTEMS

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KEYWORDS

Switched systems, Mode-independent state feedback, Switched Lyapunov function, LMI

ABSTRACT

This paper addresses the stabilization problem of discrete switched linear systems. When the mode is available, a mode-dependent state feedback controller is developed. The main contribution of this note is to provide a less conservative approach to design the mode-independent state feedback controller where the switching mode is not accessible. Both design procedures are expressed in terms of linear matrix inequalities (LMIs). In fact, the new approach provides a family of LMI parameterized by a scalar variable which makes it useful for designing a mode-independent controller and offering an additional degree of freedom. Numerical evaluation is provided to show the effectiveness of the proposed conditions.

INTRODUCTION

Switched linear systems is a class of systems that has been an attractive subject of research during the last decades since it is an appropriate class of systems to model the behavior of practical systems.They are dynamical systems simultaneously containing mixtures of logic and continuous dynamics (D.Liberzon et al.,1999; A.S.Morse,1996). A switched system is represented by a set of continuous-time or discrete-time subsystems and a rule that orchestrates the switching among them. The motivation for studying switched systems is from the fact that many practical systems are required to describe their hybrid behaviour which may depend on various environmental factors and also because the modelling of many complex systems is only possible through combination of the classical continuous laws of control with a logical discrete technique(M.Zefrani et al.,1998). Another source of motivation for studying switched systems comes from the rapidly developing area of switching control. Control techniques based on switching between different controllers have been applied extensively in recent years, particularly in the adaptive context, where they have been shown to achieve stability and improve transient response. The importance of such control methods also stems in part from the existence of systems that cannot by asymptotically stabilized by a single continuous feedback control law. Switched systems have numerous applications in control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters, and many other fields which include the modelling of communication networks, networked control systems, the modelling of bio-chemical reactions, the control of nonlinear systems that cannot be stabilized by continuous control laws, the control of systems with large uncertainty using logic-based supervisors, etc. The classical example is the case of continuous time processes that are supervised using logical decision making algorithms. In recent years, particular efforts of researches have received an increasing interest and a growing attention in the study of the stability analysis and control design for switched systems (D.Liberzon et al.,1999; Hui Ye et al.,1995).We address the stability issues and control synthesis for discrete switched systems under arbitrary switching sequences.

In fact, much efforts has been devoted to establishing analysis tools such as multiple Lyapunov functions(MLF), piecewise Lyapunov function (PLF) and switched Lyapunov function (SLF)(M.S.Branicky,1998) (D.Koutsoukos et al.,2002; R.DeCarlo et al.,2000). The approach used in this note is based in the existence of a particular quadratic Lyapunov function making full use of switching nature of the switched system, specially it has the same switching signals as the switched system. It is called SLF (Switched Lyapunov Function) which is a switching-sequence dependent Lyapunov function. It is proved in this note that with the use of switched lyapunov function less conservative results can be obtained (J.Daafouz et al.,2001; J.Daafouz et al.,2002). So, the results proposed in this work can be considered as a tradeoff between higly conservative results (those using a single quadratic Lyapunov function) and less conservative results. The stabilization problem of this class of systems has attracted a lot of researchers and interesting results were reported. For the synthesis of control switched systems with mode-independent controller limited research has been done. In fact, the results obtained using the classical quadratic approach (with a single quadratic Lyapunov function) leads to a highly con-

Proceedings 23rd European Conference on Modelling and Simulation ©ECMS Javier Otamendi, Andrzej Bargiela, José Luis Montes, Luis Miguel Doncel Pedrera (Editors)
ISBN: 978-0-9553018-8-9 / ISBN: 978-0-9553018-9-6 (CD)

servative results. Moreover, most of the existing results on the stabilization of the switched system require the critical assumption on the accesssibility of the switching mode.Practically, this assumption may sometimes be hard to satisfy and therefore, the developed results are restrictive. One alternative way is to overcome the availability of the switching mode which consists of estimating the mode and then use the existing results. On the other hand, the mode independent controller is very attractive due to its simple structure. To design such controllers, two approaches can be adopted. The first one employs a constant Lyapunov function, which makes the design problem formulated as a linear matrix inequality (LMI) problem. The second one uses mode-dependent Lyapunov function, which usually leads to less conservative results.

This note is organised as follows. In section II, we introduce the problem formulation and we give some preliminaries. Section III, stability analysis of switched systems by mean of a switched quadratic Lyapunov function is stated. Section IV covers the design of stabilizing state feedback under the assumption of the complete access to the state vector and the switching mode. On the other hand, practically, the complete access for the switching mode may not be possible for some applications and a design of mode-independent state feedback controller is of great interest in this case. A numerical evaluation is provided in section V. Finally,the paper is concluded in section VI.

NOTATION

The notations used throughout the paper are standard. The relation $A > B$ ($A > B$) means the matrix $A-B$ is positive (negative) definite. The matrix I denotes identity matrix of appropriate dimension. • is used for the blocks induced by symmetry. N is the amount of subsystems. $Conv\{\}$ stands for convex combination.

PROBLEM FORMULATION AND PRELIMINARIES

Consider linear switched system in discrete domain described by the following state equation:

$$\begin{cases} x(k+1) = A_\alpha x(k) + B_\alpha u(k) \\ y(k) = C_\alpha x(k) \end{cases} \quad (1)$$

Where

$x(k) \in R^n$ is the state vector of the system, $u(k) \in R^m$ is the control input, $y(k) \in R^p$ is the measured output, $\alpha_i(k)$ is the switching signal.

$$\alpha_i(k) : Z^+ \to \{0,1\}, \sum_{i=1}^{N} \alpha_i(k) = 1, k \in Z^+ = \{0,1,...\} \quad (2)$$

It specifies which subsystem will be activated at a certain discrete time.

Remark: The model defined by (1) naturally imposes a discontinuity on A_α since this matrix must jump instantaneously from A_i to A_j for some $i \neq j$ once switching occurs. In other words, A_α is constrained to jump among N vertices of the vector polytope $conv\{A_1, ..., A_N\}$

Furthermore, for a particular switching laws the switched system can be viewed as a special case of a time-varying linear system and therefore the usual definitions of stability can be used, see for example (R.DeCarlo et al.,2000).

We present now two useful lemmas used in proof.

Lemma 1. *(**Projection Lemma**) Given a symmetric matrix $\Psi \in IR^{m\times m}$ and two matrices P, Q of column dimension m there exists X such that the following LMI holds: $\psi + P^TX^TQ + Q^TXP < 0$*
if and only if the projection inequalities with respect to X are satisfied
$N_P^T\psi N < 0, N_Q\psi N_Q^T < 0$
where N_p and N_Q denote arbitrary bases of the nullspaces of P and Q respectively.

Proof. See (S.Boyd et al.,1994)

Lemma 2. *Let Φ a symmetric matrix and N,J matrices of appropriate dimensions.The following statements are equivalent:*

(i) $\Phi < 0$ and $\Phi + NJ^T + JN^T < 0$

(ii) There exists a matrix G such that: $\begin{bmatrix} \Phi & J+NG \\ J^T+G^TN^T & -G-G^T \end{bmatrix} < 0$

Proof. The proof is obtained remarking that: $\begin{bmatrix} \Phi & J+NG \\ J^T+G^TN^T & -G-G^T \end{bmatrix} = \begin{bmatrix} \Phi & J \\ J^T & 0 \end{bmatrix} + \begin{bmatrix} N \\ -I \end{bmatrix} G \begin{bmatrix} 0 & I \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} G^T \begin{bmatrix} N^T & -I \end{bmatrix}$
and applying lemma1.

PROBLEM

Here, we are interested in stability analysis and control synthesis problems for this class of switched system. By stability analysis, we mean stability analysis of the origin for the autonomous switched system. A method for control synthesis is provided by using switched Lyapunov function in order to find a state feedback controller. Both design procedures of mode-dependent and mode-independent state feedback are considered. Because, the problem of estimation of the switching times makes the operation of detection mode difficult. Switched systems are systems subject to abrupt changes in their structure and parameters. They are well suited to represent dynamic systems subject to switches between alternative configurations. This kind of systems represents some limitation in the identification of the operating mode. So, finding a fixed controller is in this case interesting. A method for control synthesis is provided by using switched Lyapunov function in order to find a mode-independent state feedback control.

$$u(k) = Kx(k) \tag{3}$$

ensuring stability of the closed loop switched system

$$x(k+1) = (A_\alpha + B_\alpha K)\, x(k) \tag{4}$$

STABILITY ANALYSIS

In this section, we address the problem of stability analysis of the origin of an autonomous switched system given by:

$$x(k+1) = A_\alpha x(k) \tag{5}$$

Define the indicator function:

$\alpha(k) = [\alpha_1(k), ..., \alpha_N(k)]^T$.

Then, the switched system can also be written as:

$$x(k+1) = \sum_{i=1}^{N} \alpha_i(k) A_i x(k) \tag{6}$$

When:

$$\alpha_i(k) = \begin{cases} 1 \text{ when the ith mode Ai is activated} \\ 0 \text{ otherwise} \end{cases}$$

To check stability of the switched system, we use this switched Lyapunov function defined as:

$$V(k, x(k)) = x(k)^T P(\alpha(k)) x(k) \tag{7}$$

$$V(k, x(k)) = x(k)^T \left(\sum_{i=1}^{N} \alpha_i(k) P_i \right) x(k) \tag{8}$$

With $P_1, ..., P_N$ symmetric positive definite matrices. If such a positive definite Lyapunov function exists and $\Delta V(k, x_k) = V(k+1, x_{k+1}) - V(k, x_k)$ is negative definite along the solution of (6) then the origin of the switched system given by (5) is globally asymptotically stable as shown by the following general theorem.

Theorem 3. *The equilibrium 0 of (J.Daafouz et al.,2002)*

$$x(k+1) = f_k\left(x(k)\right) \tag{9}$$

Is globally uniformly asymptotically stable, if there exists a function $V : Z^+ \times R^n \to R$ such that:

• V is a positive definite function, decrescent and radially unbounded

• $\Delta V(k, x_k) = V(k+1, x_{k+1}) - V(k, x_k)$ is negative definite along the solution (6)

The Lyapunov function (8) is a positive definite function, decrescent and radially unbounded since

$$V(k, 0) = 0 \; \forall \; k \geq 0 \tag{10}$$

And

$\beta_1 \left\| x_k \right\|^2 \leq V(k, x_k) = x_k^T (\sum_{i=1}^{N} \alpha_i(k) P_i) x_k \leq \beta_2 \left\| x_k \right\|^2$

For all $x(k) \in R^n$ and $k \geq 0$ with $\beta_1 = \min_{i \in I} \lambda_{\min}(P_i)$ and $\beta_2 = \max_{i \in I} \lambda_{\max}(P_i)$ positive scalars.

Proof. See (Vidyasagar,1993)

MAIN RESULT

In the following theorem, we give four equivalent necessary and sufficient conditions for the existence of the Lyapunov of the form (8) whose difference is negative definite, proving asymptotic stability of (6).

Theorem 4. *The following statements are equivalents*

(i) There exists a Lyapunov function of the form (8) whose difference is negative definite, proving asymptotic stability of (6)

(ii) There exist N symmetric matrices $P_1, ..., P_N$ satisfying

$$\begin{bmatrix} -P_i & A_i^T P_j \\ P_j A_i & -P_j \end{bmatrix} < 0 \; \forall \; (i, j) \in I \times I \tag{11}$$

The Lyapunov function is then given by:

$V(k, x(k)) = x(k)^T \left(\sum_{i=1}^{N} \alpha_i(k) P_i \right) x(k)$

Ie: $\Delta V(k, x_k) = 0 \forall \; k \geq 0$ and $\Delta V(k, x_k) \leq -\gamma(\|x_k\|) \forall k \geq 0 \; \forall \; x_k \in R^n$ where γ is of class K.

[A function $\gamma : [0, \infty) \to [0, \infty)$ is of class K if it is continuous, strictly increasing, zero at zero and unbounded ($\gamma(s) \to \infty$ as $s \to \infty$)]

(iii) There exist N symmetric matrices $S_1, ..., S_N$ and N matrices $G_1, ..., G_N$ satisfying

$$\begin{bmatrix} -G_i - G_i^T + S_i & G_i^T A_i^T \\ A_i G_i & -S_j \end{bmatrix} < 0 \; \forall \; (i, j) \in I \times I \tag{12}$$

The Lyapunov function is then given by:

$V(k, x(k)) = x(k)^T \left(\sum_{i=1}^{N} \xi_i(k) S_i^{-1} \right) x(k)$

(iv) There exist N symmetric matrices $S_1, ..., S_N$ and N matrices $G_1, ..., G_N$ satisfying

$$\begin{bmatrix} \alpha_i^2 S_i - S_j & A_i G_i + \alpha_i G_i - \alpha_i S_i \\ \bullet & S_i - G_i - G_i^T \end{bmatrix} < 0 \; \forall \; (i, j) \in I \times I \tag{13}$$

α_i being arbitrary and $\alpha_i \in]-1, 1[$.

The Lyapunov function is then given by:

$V(k, x(k)) = x(k)^T \left(\sum_{i=1}^{N} \alpha_i(k) S_i^{-1} \right) x(k)$

Proof.

• To prove $(i) \Leftrightarrow (ii) \Leftrightarrow (iii)$ see (J.Daafouz et al.,2002)

• To prove $(ii) \Leftrightarrow (iv)$ assume that:

$$\begin{bmatrix} -S_i & S_i A_i^T \\ \bullet & -S_j \end{bmatrix} < 0$$

Which can be written as:

$$\begin{bmatrix} -S_i & S_i A_i^T \\ \bullet & -S_j \end{bmatrix} =$$

$$\begin{bmatrix} -S_i & -\alpha_i S_i \\ -\alpha_i S_i & -S_j \end{bmatrix} + \begin{bmatrix} S_i \\ 0 \end{bmatrix} \begin{bmatrix} 0 & A_i^T + \alpha_i I \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ A_i + \alpha_i I \end{bmatrix} \begin{bmatrix} S_i & 0 \end{bmatrix} < 0$$

with $\Phi < 0$, for all $\alpha_i \backslash -1 < \alpha_i < 1$.
By applying lemma2, this equivalent to the existence of matrices G_i such that:

$$\begin{bmatrix} -S_i & -\alpha_i S_i & S_i \\ -\alpha_i S_i & -S_j & A_i^T G_i + \alpha_i G_i \\ S_i & G_i^T A_i^T + \alpha_i G_i^T & -G_i - G_i^T \end{bmatrix} < 0$$

Which is equivalent by the schur complement to:

$$\begin{bmatrix} -S_j & A_i G_i + \alpha_i G_i \\ \bullet & -G_i - G_i^T \end{bmatrix} + \begin{bmatrix} -\alpha_i S_i \\ S_i \end{bmatrix} S_i^{-1} \begin{bmatrix} -\alpha_i S_i & S_i \end{bmatrix} < 0$$

$$\begin{bmatrix} \alpha_i^2 S_i - S_j & A_i G_i + \alpha_i G_i - \alpha_i S_i \\ \bullet & S_i - G_i - G_i^T \end{bmatrix} < 0$$

Remarks:

- In particular, for $\alpha_i = 0$, we found the results developed in (J.Daafouz et al.,2002)
- for $\alpha_i \in]-1, 1[$, we obtain a family of LMIs conditions parametrized by α_i

MODE-DEPENDENT STATE FEEDBACK DESIGN

Consider the synthesis of a switched static mode-dependent state feedback.

$$u(k) = K_\alpha x(k) \quad (14)$$

guaranting stability of the closed-loop switched system (6). This problem reduces to find P_i and K_i ($\forall\, i \in I$) , such that:

$$\begin{bmatrix} -P_i & (A_i + B_i K_i)^T P_j \\ P_j (A_i + B_i K_i) & -P_j \end{bmatrix} < 0 \quad (15)$$

$\forall (i, j) \in I \times I$ Or equivalently to find S_i, G_i and K_i Such that

$$\begin{bmatrix} -G_i - G_i^T + S_i & G_i^T (A_i + B_i K_i)^T \\ (A_i + B_i K_i) G_i & -S_j \end{bmatrix} < 0 \quad (16)$$

$\forall (i, j) \in I \times I$

Theorem 5. *If there exist symmetric matrices S_i, matrices G_i and R_i ($\forall\, i \in I$) such that $\forall\, (i, j) \in I \times I$*

$$\begin{bmatrix} S_i - G_i - G_i^T & (A_i G_i + B_i R_i)^T \\ A_i G_i + B_i R_i & -S_j \end{bmatrix} < 0 \quad (17)$$

Then the mode-dependent state feedback given by (14) with

$$K_i = R_i G_i^{-1}\, \forall\, \mathrm{i} \in \mathrm{I} \quad (18)$$

asymptotically stabilizes (1).

- The next section shows that the contribution of Theorem 4 is to propose a condition which is less conservative for the problem of a Mode-Independent state feedback.

MODE-INDEPENDENT STATE FEEDBACK CONTROLLER

The results of the previous section assume the complete access of the switching mode. Practically, this may not be possible for some application and a design of mode-independent state feedback controller is of great interest in this case. We now seek a stabilizing mode-independent control law given by:

$$u(k) = Kx(k) \quad (19)$$

where K $\in R^{n_u \times n}$ is a constant matrix. This approach may be restrictive compared to mode-dependent state feedback controller but has less conservative to what has been reported in the literatures(those using a single quadratic Lyapunov function).
The following theorems provide conditions for finding a mode-independent state feedback controller.

Theorem 6. *If there exist symmetric matrix P, a matrix R Such that*

$$\begin{bmatrix} -P & (A_i P + B_i R)^T \\ \bullet & -P \end{bmatrix} < 0\, \forall\, i \in I \quad (20)$$

Then the mode independent state feedback controller is given by:

$$K = RP^{-1} \forall\, \mathrm{i} \in \mathrm{I} \quad (21)$$

Theorem 7. *If there exist symmetric matrices S_i, a matrix G, a matrix R and arbitrary prescribed numbers $\alpha_i \in]-1, 1[$ such that:*
$\forall\, (i, j) \in I \times I$

$$\begin{bmatrix} \alpha_i^2 S_i - S_j & A_i G + B_i R + \alpha_i G - \alpha_i S_i \\ \bullet & S_i - G - G^T \end{bmatrix} < 0 \quad (22)$$

Then the mode independent state feedback controller is given by:

$$K = RG^{-1}\, \forall\, \mathrm{i} \in \mathrm{I} \quad (23)$$

Theorem 8. *Letting $R_i = KG_i\ \forall\ \mathrm{i} \in \mathrm{I}$ and $G_i = \alpha_i G\ \forall \mathrm{i} \in \mathrm{I}$, If there exist symmetric matrices S_i, matrices G_i, matrices R_i and arbitrary prescribed numbers $\alpha_i \in]-1, 1[$ such that*
$\forall (i, j) \in I \times I$

$$\begin{bmatrix} \alpha_i^2 S_i - S_j & \alpha_i A_i G + \alpha_i B_i K G + \alpha_i^2 G - \alpha_i S_i \\ \bullet & S_i - \alpha_i G - \alpha_i G^T \end{bmatrix} < 0 \quad (24)$$

Then the mode independent state feedback controller is given by:

$$K = RG^{-1}\, \forall\, \mathrm{i} \in \mathrm{I} \quad (25)$$

NUMERICAL EVALUATION

In this section, a numerical evaluation is proposed. The problem considered here is the design of a static mode-independent state feedback controller stabilizing the switched system. The result obtained using the theorem (8) is compared to the three methods given by the theorems (5), (6), (7) and summarize in the table 1. The switched system is characterized by: the number of modes (N), the system order (n), number of inputs (m) and the number of outputs (p). For fixed values of (N, n, m, p), we generate randomly 100 switched systems of the form (1). So the purpose is to design by using the four methods a feedback controller in the form (2) such that the closed -loop system is asymptotically stable.

- Method 1: uses constant Lyapunov function (CLF)$V(x_k) = x_k^T P x_k$.

and applying the conditions given in Theorem 6.

- Method 2: uses the conditions given in Theorem 7.
- Method 3: uses the conditions given in Theorem 8.
- Method 4: uses the conditions given in Theorem 5.

By using the matlab LMI Control Toolbox to check the feasibility of the LMI conditions, we introduce a counter (Success Method1, Success Method2, Success Method3 and Success method4) which is increased if the corresponding method succeeds in providing an state feedback stabilizing control.

Table 1: Numerical Evaluation

Switched System	Success	N=2	N=3
n=3	Method 1	97	88
m=1	Method 2	100	96
p=1	Method 3	100	98
	Method 4	100	100
n=4	Method 1	91	77
m=1	Method 2	100	82
p=1	Method 3	100	86
	Method 4	100	100
n=5	Method 1	90	49
m=1	Method 2	98	64
p=1	Method 3	98	64
	Method 4	100	100
n=4	Method 1	100	97
m=2	Method 2	100	99
p=1	Method 3	100	99
	Method 4	100	100
n=5	Method 1	100	93
m=2	Method 2	100	96
p=1	Method 3	100	96
	Method 4	100	100
n=4	Method 1	94	77
m=1	Method 2	100	84
p=2	Method 3	100	84
	Method 4	100	100
n=5	Method 1	88	46
m=1	Method 2	97	56
p=2	Method 3	97	62
	Method 4	100	100
n=7	Method 1	96	36
m=2	Method 2	100	83
p=1	Method 3	100	89
	Method 4	100	100

SUMMARY AND CONCLUSIONS

In this paper, the problem of stability and synthesis of discrete switched linear systems using switched Lyapunov function (SLF) is investigated. Tow types of controllers were designed using the LMI setting. The first one assumes the complete access to the switching modes which gives mode-dependent controller. The second one relaxes this assumption and a mode-independent gain is obtained which may be restrictive compared to mode-dependent state feedback controller but has less conservatism to what as been reported in the literatures. The derived conditions, are expressed as a family of linear matrix inequalities (LMIs)parameterized by the scalar variables α_i. These conditions reduce significantly the conservatism and show the advantage of using the scalar variables in the case of mode-indepent state feedback. Numerical evaluations are given to demonstrate the applicability and the conservatism reduction of the proposed conditions and a comparison with recent conditions proposed in the literature has been described.

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