

variance in output values, so only the most influential parameters should be described by probability distributions.

Step 1.2 Fitting the probability distribution

After making the decision that a particular input parameter should be treated as uncertain, we have to fit a probability distribution: empirical or theoretical one, discrete or continuous. This step involves data analysis, expert judgment, and literature review.

Step 2 Selecting outputs

Simulation usually produces probability distributions for more than one output. One of the more essential advantages of simulation modelling is the ability to consider many output parameters.

Step 3 Running singular simulation experiment

Step 3.1 Generating random numbers

In order to obtain the changes consistent with the selected distribution, two stages must be accomplished: (1) generation of uniformly distributed random variates between 0.0 and 1.0, and (2) transformation of the uniformly distributed random variates into random variates from other distribution, both continuous and discrete. In Excel, the first stage may be accomplished with *RAND()* function, a random number generator. It is recommended to use separate uniformly distributed streams to model every random variable defined in the model.

Step 3.2 Generating random variates

Formulas to convert uniformly distributed variates into random variates from other distributions are usually (but not always) accessible in standard version of spreadsheet software.

Step 3.3 Constructing the formulas

Analytical model connects input data (deterministic and stochastic) with output measures through logical formulas.

Step 3.4 Running n independent replications (trials) ($n \rightarrow \infty$)

Executing one single simulation experiment consists of sampling n values from input distributions, introducing the samples into logical formulas, calculating output values and storing the results of the outputs. The key choice is how many samples to draw – the more samples we take, the more precise our estimates become.

Step 4 Running the full simulation study

Full simulation experiment may sometimes require repeating the single experiment k times (to run n replications k times). This approach is recommended when observing the *typical* behaviour of the system and will guarantee the better precision.

Step 5 Output analysis

The final step in the simulation analysis involves using the output data to estimate or graphically show the output parameters of the model. Usually the histograms

are drawn, standard deviations and a 90 or 95 percent confidence intervals for the mean are computed. Sometimes an optimal set of decisions is identified through properly arranged what-if calculations.

WHY TO TEACH MONTE CARLO SIMULATION TO MANAGEMENT STUDENTS

Monte Carlo approach is categorized as the static, stochastic simulation. It is the method of random sampling delivering the approximate solution. In MC models time does not play the essential role, though it may be used to observe the changes in the systems' states. This dynamic face of MC method is visible particularly in financial modelling.

Monte Carlo simulation is usually done with spreadsheets. This transfers from spreadsheet modelling onto MC approach the useful features, which offer the additional benefits when introducing simulation modelling approach to management and business students. The main, crucial advantages of Monte Carlo modelling approach is given below.

Common knowledge about spreadsheets

Spreadsheets play the crucial role in business analysis and they are commonly used in academic studies. Students entering the simulation modelling courses are in majority familiar with MS Excel and have basic Excel skills. It allows the academic teacher to start the simulation course directly with *simulation* knowledge, without necessity of introducing the new computer tools. Though, usually, the discrete-event dedicated software like Extendsim, Arena, GPSS or others will be obligatorily demonstrated in further part of the course, the introductory part may be entirely dedicated to stochastic simulation basics.

The additional favourable circumstance is the access to licensed version of the spreadsheet software, which is the common standard in academic laboratories and, even more important – in business practice. The future manager will probably prefer to practice the simulation methods with at-hand tool, than to look after sophisticated software which may not be available at once.

Uncomplicated structure of the models

The nature of static simulation causes the models to have simple structures compare to dynamic models. Less complicated models allow to focus on the nature of the stochastic simulation modelling rather than to pay attention on the multiple model elements. Monte Carlo model is composed of few logical expressions, which are replicated in the succeeding rows (trials) according to the assumed number of replications. This gives the “end-user modellers” (and this applies to management students) who are not specialists in modelling and software programming, the possibility to learn more about uncertain simulation mechanisms. The model answers are clearly visible immediately after the simulation run is completed.

Broad range of applications

Simulation model is a *mean* to achieve the goal and not *the goal* itself. The overall aim of the simulation modelling course is to teach the students the skills to formulate the questions referring to system's behaviour, to use the modelling skills to observe the system and to recognize the right answers. Only then the tools, techniques and methods may be introduced. The broad range of Monte Carlo models' applications makes it possible to plan the academic course according to the student's profile. MC models have been employed for the solution of different problems in finance (option-pricing, cash-management, risk estimation, setting the optimal investment strategies) and management (inventory models for stock control, allocation models for scheduling production, what-if planning, project management). The authors used Monte Carlo approach in teaching process to work out the models: of production line processing, of clients' being serviced in work centre, of tickets' sale for mass sporting event, of communication between client and on-line shop, of planning the optimal investment stock-market portfolio, of identifying the uncertain disturbances in sewage treatment plant, and many others.

Transparency of MC simulation process

Based on the Monte Carlo models, the difficult and non-intuitive issues related to the probabilistic behaviour of the phenomena under studies, as well as the strategy of inserting uncertainties into simulation model, can be described and explained in comprehensive and convincing way.

The consequences of manipulation with uncertainties are instantly visible in spreadsheet. Uncomplicated analytical model does not shade the simulation process, and the relations between probabilistic input and output data are easily understandable.

In spreadsheet, the formulas are inserted manually. It is not always possible to call the ready-to-use built-in function. Sometimes student has to construct the formula, for example for generating discrete variates and a number of continuous random variates. It helps to study the random mechanisms more deeply. Generated random values along with partial output data are incessantly presented on the screen. When running the simulation experiment, student observes not only the mean output values, but also the random changes influencing the output measure. This is a good encouragement to appreciate the need for proper statistical analysis of simulation output.

The benefits of applying the variance reduction techniques to control the randomness in simulation experiment are visible *in the course* of simulation run. Also, when running the experiment, the number of replications is set-up through the continuous observation of half width intervals.

Easiness of integrating Monte Carlo simulation with other Operations Research techniques

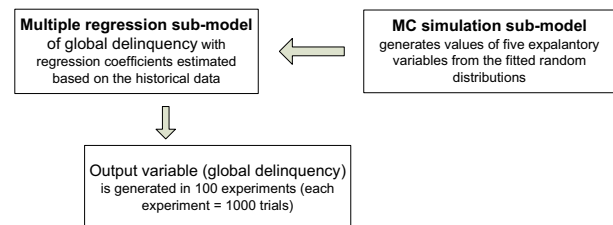
There are many specific models and many specific approaches to these models, but the real challenge is to train students in *solving the management problems*, not only in applying the certain method. Monte Carlo simulation enables a more general approach to the model-building process as it may be easily combined with other decision support techniques. The next section presents three exemplary spreadsheet decision models, exploring the relationships between the inputs and outputs and enhancing the problem analysis by the use of Monte Carlo simulation to observe the uncertain parameters.

MONTE CARLO SIMULATION MODEL WITH ECONOMETRIC MODULE (FORECASTING)

Basic Idea of the Model

Monte Carlo simulation may be used to extend the abilities of forecasting models. Once the multiple regression model is created, a dependent variable may be predicted by random sampling the values of the independent variables.

The model described below was developed by the diploma student (Otczyk 2008), under supervision of one of us (Bożena Mielczarek). The purpose of the model was to analyse trends in delinquency caused by the global economic situation. The model is divided into two sub-models (fig.1).



Figures 1: An overview MC-econometric model

The multiple regression sub-model explores the relationship of delinquency to few independent variables, namely the unemployment rate, the purchase tendencies, monthly and annual inflation, and industrial production. The second part uses Monte Carlo approach to simulate the values of five input variables. The computer experiments are aimed to forecast the level of delinquency based on the changes in economic situation expressed by five selected independent variables.

Regression Sub-Model

The multiple regression model of global delinquency is a linear function of the explanatory variables (1):

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_5 X_5 + \alpha_6 \quad (1)$$

Where:

Y – delinquency in general

X_1 – unemployment rate

X_2 – purchase tendencies

X_3 – monthly inflation
 X_4 – annual inflation
 X_5 – global industrial production
 $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ – the regression coefficients estimated using the principle of least squares, based on the historical data from ten-year period.

Monte Carlo Sub-Model

Monte Carlo sub-model generates input values of $X_1, X_2, X_3, X_4,$ and X_5 from random distributions fitted from the historical data:
 unemployment rate (X_1) – Uniform,
 purchase tendencies (X_2) – Triangular,
 monthly (X_3) and annual (X_4) inflation – Gamma,
 industrial production (X_5) - Normal distribution.
 Then, 100 experiments, each consisting of 1000 replications are run (fig.2).

replications	X1	X2	X3	X4	X5	Y
1	17,372968	92,8222326	5,275524018	0,868063583	-0,51	115524
2	19,052572	89,4712078	22,24260243	-0,799788384	10,20	111043
3	13,847358	82,486274	13,38406946	-0,613736737	6,51	114243
4	13,435487	99,8691001	15,41555321	-3,056473699	-4,48	71303
5	12,453743	86,4859363	18,5223144	-0,013436382	-0,04	96994
6	18,743393	95,5907307	13,75290594	0,428147596	10,43	114588
7	13,302376	84,1860654	7,977710084	1,374581527	-0,89	101045
8	13,17024	90,3994505	15,7096856	-0,851143002	-1,22	101445
9	11,466087	80,8703394	17,60337909	-0,999595149	-0,31	106042
10	14,97582	97,2216506	10,69784958	-0,953814722	-3,41	102255
11	20,061489	101,630632	6,961981853	1,299684638	4,51	101696
12	10,414967	102,163292	9,85378149	0,896096364	10,39	93606
13	11,928541	90,5088739	8,449160305	1,034048521	3,67	99474
14	20,737524	99,9926379	11,63364686	-0,846472747	4,98	111553
15	13,759727	86,5817348	13,09093932	0,936950866	-1,98	96884
16	16,810564	93,3925646	6,33518539	3,466182872	2,98	91268
17	15,088927	92,685303	6,630084499	-0,976932514	7,36	117056
18	14,892013	94,8033477	12,81662662	-0,056490196	-4,80	96049
19	12,990861	87,3714638	11,30273176	-0,777226368	4,79	111237
20	13,785383	96,7538225	6,95988239	-0,507082293	-6,28	100059
21	16,78375	80,3706557	3,953864507	0,081575847	1,50	120473
22	15,748972	89,971145	20,14421825	-0,768974852	1,85	102545
23	17,618698	79,0992859	8,835741028	-0,632445906	-5,77	116807
24	12,123998	102,551038	14,00310794	-0,586516424	-3,74	89700
25	20,321607	84,9683183	11,37713298	-0,950692597	1,40	120324
26	9,0392378	89,157394	16,11557309	-0,923173846	-14,28	88216

Figures 2: A Fragment of Step 3.4 of MC Simulation Module Presenting 26 of 1000 Replications

One replication generates five random numbers with *RAND()* formula, then transforms random numbers into adequate random variates and calculates value of delinquency (Y) from multiple regression equation (1). The mean value of Y is estimated from 1000 independent replications and the procedure is repeated 100 times. It means that one full simulation experiment consists of 100 singular experiments, and one singular experiment requires 1000 replications. The output mean value of Y and the confidence interval is calculated based on 100 singular experiments.

What-if Calculations

What-if calculations are aimed on the observations of the changes in delinquency, being the result of the macro-changes in five input variables. It is shown, through the simulation experiments, that macro-

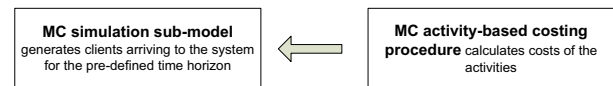
economic changes have strong influence on such social phenomena as global delinquency.

MONTE CARLO SIMULATION MODEL WITH QUEUING MODULE AND ACTIVITY-BASED COSTING APPROACH

Basic Idea of the Model

The following model is inspired by the next-event approach in queuing system simulation and the need to acquaint the students with activity-based costing technique (fig.3).

The first stage of activity-based costing procedure, c.f. (Garrison and Noreen 1994), consists in assigning the costs to activity centres (i.e. firm's departments or the groups of activities), where they are accumulated until being transferred onto the products. Costs may be assigned directly or by the means of cost drivers. The second stage assigns the costs to the products, taking into account two factors: easiness of collecting data related to cost drivers and usefulness of cost drivers as the measures of activities' engagement and utilization by the product.



Figures 3: An overview MC-queueing model activity based costing approach

Monte Carlo Sub-Model

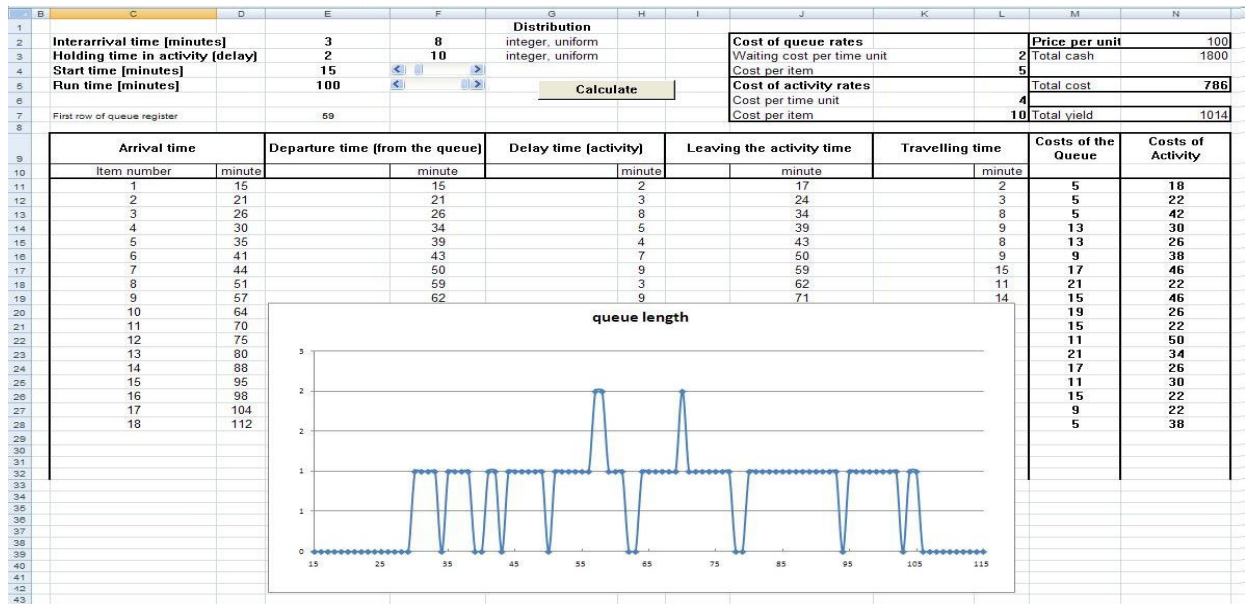
One server queueing model with activity-based costing module (fig.4) is run with input values generated from uniform distributions (inter-arrival time and holding time). The total yield is calculated according to the equation (2):

$$TotalYield = FinishedUnits * Price - TotalCost \quad (2)$$

A number of independent replications is then run and the output statistical measures are calculated.

ABC-Module

The total cost is calculated according to activity-based-costing approach. There are two groups of cost: the cost of queue rates, which is calculated if the product item is waiting in the queue, and the cost of activity rates related to the resource usage. Two coefficients (*waiting cost per time unit* and *cost per item*) are needed to calculate the first type of cost and two others (*cost per time unit* and *cost per item*) are used to estimate the second variable. The calculations are performed constantly during the simulation run.



Figures 4: Steps 1 and 3 of Monte Carlo Method. Core Queue Model. One Server Version

Model Outputs

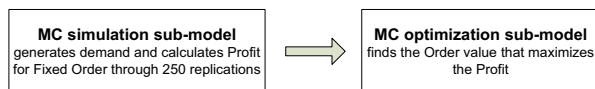
The aim of the simulation experiment is to observe the queue during the simulated process and to study the profitability of different variants in clients' servicing modes, with the use of activity-based costing approach.

MONTE CARLO SIMULATION MODEL WITH OPTIMIZATION MODULE

Basic Idea of the Model

The model is based on the example presented originally by Winston and Albright (1997). Simulation does not belong to optimization techniques but the simulation experiments in spreadsheet may be planned to find the *best* value of decision variable, i.e. a value that maximizes (minimizes) the selected output variable (fig.5).

A company plans a purchase of a product (Valentine cards) to meet customer demands. If they order too few items, they will lose potential profit. If they order too many – they will have to store cards, which were not sold, for the next year.



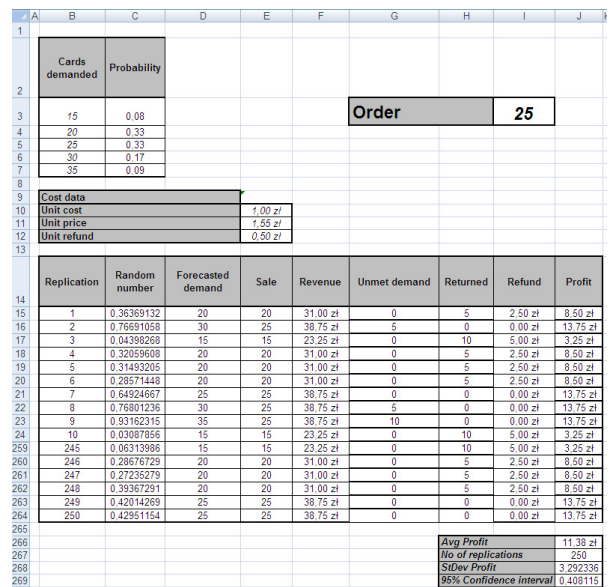
Figures 5: An Overview of MC Optimization Model

Monte Carlo Sub-Model

First (fig.6), demand for Valentine cards is generated according to discrete (empirical) probability distribution. Then the profit is calculated according to the formula (3) for a *fixed* order quantity:

$$Profit = Revenue + Refund - OrderingCost \quad (3)$$

Each *Profit* value corresponds to one of 250 randomly generated demand and it illustrates an independent replay of the company's problem. The average value and standard deviation of 250 profits values is calculated below.



Figures 6: MC Simulation Module. Valentine Cards

Optimization Sub-Model

Optimization sub-model investigates the way the profit depends on the order quantity (fig.7). Optimization experiment are run 100 times separately for different *order* quantities. Each cell in the table on fig.7 shows the average profit calculated based on 250 replications of MC model (fig.6) for a particular order quantity.

The summarized statistics allow to point the best order quantity – the quantity which maximizes the profit.

	L	M	N	O	P	Q	R	S	T	U
Replication	Order quantity									
11,38 zł	15	20	21	22	23	24	25	30	35	
1	8.25	10.685	10.7268	10.987	10.9784	10.7934	10.789	10.242	8.141	
2	8.25	10.727	10.6932	10.9618	10.949	11.268	11.314	9.99	8.582	
3	8.25	10.517	10.6638	10.6846	10.991	11.0832	11.146	9.969	7.616	
4	8.25	10.643	10.773	10.9324	10.8188	10.9362	11.167	10.053	8.435	
5	8.25	10.475	10.605	10.9534	11.2136	10.8984	11.734	9.423	8.246	
6	8.25	10.475	10.6302	10.8694	10.7852	11.2302	11.419	10.053	8.309	
7	8.25	10.601	10.9242	10.7812	11.138	11.2638	11.713	9.633	8.309	
8	8.25	10.412	10.731	10.8148	11.0498	11.269	11.062	9.57	7.91	
9	8.25	10.664	10.7604	10.6384	11.0414	11.3856	10.894	10.389	8.015	
10	8.25	10.685	10.731	10.8022	10.8188	11.0496	11.314	10.011	7.994	
95	8.25	10.517	10.7058	10.588	10.8188	11.3016	10.81	9.948	7.994	
96	8.25	10.559	10.5714	10.8148	11.117	10.911	10.915	9.864	7.805	
97	8.25	10.622	10.584	10.9072	10.7726	11.0118	11.44	10.116	7.973	
98	8.25	10.685	10.7394	10.6972	11.1632	10.8438	11.272	9.927	8.078	
99	8.25	10.538	10.689	11.0626	11.0456	11.1084	11.146	9.864	8.435	
100	8.25	10.706	10.71	10.8022	10.7642	11.3604	11.293	9.717	8.036	
	15	20	21	22	23	24	25	30	35	
Average	8.25	10.579738	10.69336	10.81821	10.94485	11.08063	11.21	9.99945	8.010275	
No of replications	100	100	100	100	100	100	100	100	100	
Std Dev	0	0.089189	0.11095	0.141968	0.148451	0.1725	0.194472	0.265061	0.39818	
95% Conf Interval		0.0174807	0.021746	0.027806	0.029102	0.033829	0.038116	0.051951	0.078042	

Figures 7: Optimization Module. Valentine Cards

SIMULATION LABORATORY COURSE

The typical laboratory course on *simulation modeling* usually starts with repetition of basic statistical definitions. Then the simple “on-hand” simulation model in spreadsheet is run and the basic Monte Carlo concepts are introduced: the formula for random numbers generation *RAND()*, the technique for discrete random variates generation through *VLOOKUP()* formula, the method to transform random numbers into a range of continuous distributions (*Uniform, Triangular, Exponential*), the Excel formula to generate normal variates *NORMINV(...)*, the importance of confidence interval calculations and the choice of adequate number of replications. Finally, the concept of applying *Data/Table* command is introduced. The command enables to change the value of decision variable and to compare the selected performance measure in repeated simulation experiments.

A range of simulation spreadsheets models may now be introduced to illustrate the usefulness of simulation approach in solving decision management problems. At this point, the exemplary models integrating Monte Carlo method with other decision solving techniques may be discussed to show the students the flexibility and versatility of spreadsheet simulation modeling.

FINAL REMARKS

Spreadsheets are the principal vehicle for modelling in business and despite some limitations they are willingly used in teaching process in logistics, management, finance, marketing, engineering and operations research. Spreadsheet-based Monte Carlo simulation models may support wide range of decision problems (Schriber 2009) and this methodology may be easily adopted by management practitioners. Spreadsheet-based approach is also the recognized method, c.f. (Pecherska and Merkurjev 2005) to

present the basic concepts of static stochastic, discrete-event and System Dynamic simulations.

This paper describes the benefits of integrating Monte Carlo approach with other decision support techniques to create spreadsheet-based models to be used in teaching process of management studies.

The paper examines the spreadsheet based Monte Carlo simulation technique and its elasticity when integrating with other modeling methods. This feature enhances the value and practical applicability of Monte Carlo simulation as a decision support tool in management practice.

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