SIMULATION OF INVENTORY CONTROL SYSTEM FOR SUPPLY CHAIN "PRODUCER – WHOLESALER – CLIENT" IN EXTENDSIM ENVIRONMENT

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Inventory control, supply chain, wholesaler, customer, simulation, ExtendSim package

ABSTRACT

There is considered a two-level single-product inventory control system which controls correspondingly the wholesale's warehouse and the customers' warehouses. For delivering the product there has been organized a supply chain "producer – wholesaler – customer". It is assumed that the customers and the wholesaler shape their stocks having in mind the minimization of the total costs of the product ordering, holding and losses from deficit per time unit. The customers' demands for the product and the time of delivering the cargo from the producer to the wholesaler and from the wholesaler to each of the customers are random values with known laws of distribution. The wholesaler orders the goods at the fixed, equally distant, moments of time. The order of goods is performed by the customer at a random moment of time, when the remains of the goods in his warehouse have reduced up to a fixed level called the reorder point. In the given paper there is suggested a simulation model of the above inventory control. The ExtendSim 8 package has been used as the means of simulation. The numerical examples of the problem solving are presented.

INTRODUCTION

In the given paper we consider a stochastic single-product inventory control model for the chain "producer – wholesaler – customer" (Chopra and Meindl 2001; Magableh and Mason 2009). In practice there are many cases when a wholesaler is involved in ordering process and a situation "producer – wholesaler – customer" takes place. We have to take into account that the sum of total costs for goods ordering, holding and losses from deficit per time unit should be minimal. In proposed criteria total costs are sum of corresponding costs (losses) for all subjects taking part in the ordering process, in our case for customers and a wholesaler.

The wholesaler and customers can use different ordering strategies. Two models of ordering process used by customers were considered in the authors' work (Kopytov et al. 2007). The first model is a model with fixed reorder point and fixed order quantity, second model is the model with fixed period of time between the moments of placing the neighboring orders. The wholesaler's ordering algorithms were presented in the paper (Kopytov et al. 2005). The simplest algorithm considers the following situation: every customer's order is sent by the wholesaler to the producer at once and received goods are sent back to the customers at once too. In the second variant the wholesaler constructs a common order for group of customers taking in account or an inventory level for each customer, or a time of receiving the customers' orders and a quantity of them. In the given paper we have proposed the third variant when the wholesaler has his own warehouse with the definite quantity of goods. In practice suggested models are realized using analytical and simulation approaches. In the given paper the considered task is solved using a simulation method.

DESCRIPTION OF THE MODEL

Let us consider a single-product inventory system for the supply chain "producer – wholesaler –customer" with two stages in ordering process (see Fig.1).The *f*irst stage is executed by n customers. In the moment of time, when the customer's stock level falls to a certain level, a new order is sent to the wholesaler. The second stage is executed by the wholesaler. A new order is sent to the producer in the fixed moments of time. We assume that the wholesaler has his own warehouse.

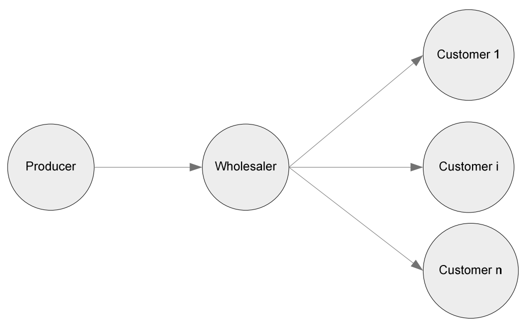

Figure 1: Chain of Product Ordering

The principal aim of the considered problem is to define the exact ordering strategy for n customers and the wholesaler to achieve the minimum expenses in inventory control system per time unit (Kopytov et al. 2007). Taking into account stochastic character of the inventory control problem, the criterion of optimization

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is minimum average total expenses E^{total} per time unit, which are calculated by following formula:

$$E^{\text{total}} = \sum_{i=1}^{n} E_i^{\text{cust}} + E^{\text{wh}}, \qquad (1)$$

where E_i^{cust} is i-th customer's average total expenses for goods holding, ordering and losses from deficit per time unit, $i = 1, 2, ..., n$; E^{wh} is wholesaler's average total expenses for goods holding, ordering and losses from deficit per time unit. Let us consider in detail the stages in the presented chain of product ordering.

First stage of ordering process

The demand for goods D_i of i-th customer is Poisson process with intensity λ_i. Time L_i of goods delivery from the wholesaler to i-th customer has a normal distribution with parameters μ_i and σ_i. The policy of order forming for i-th customer is as follows: A new order is placed at the moment of time, when the stock level falls to certain level R_i. The order quantity Q_i is constant. We suppose that $Q_i \geq R_i$. Note that order reorder point R_i and quantity Q_i are control parameters of the first stage model. Dynamics of the inventory level of product for i-th customer during one cycle $T_i = T_i^* + L_i$ (time interval between two neighboring order deliveries for i-th customer) is shown in Fig.2. There is a possible situation of deficit, when demand $D_i(L_i)$ during lead time L_i exceeds the reorder point R_i. We suppose that in case of deficit the last cannot be covered by expected order.

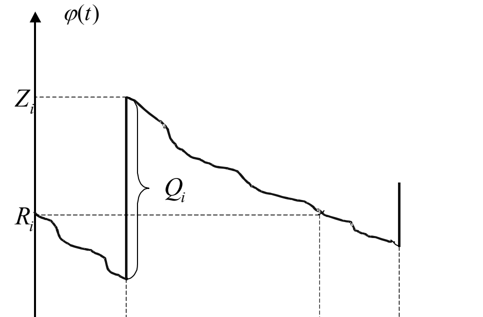

Figure 2: Dynamics of i-th Customer's Inventory Level During One Cycle

Denote as Z_i the quantity of goods in stock in the time moment immediately after order receiving. This random variable Z_i is determined as a function of demand $D_i(L_i)$ during lead time L_i:

$$Z_i = \begin{cases} R_i + Q_i - D_i(L_i), & \text{if } D_i(L_i) < R_i; \\ Q_i, & \text{if } D_i(L_i) \geq R_i. \end{cases} \qquad (2)$$

Formula (2) allows expressing different economical indexes of considered process. We suppose that the wholesaler has his own warehouse with a definite quantity of goods q. If customer's order quantity is less or equal than quantity of products in the stock ($Q_i \leq q$) the wholesaler performs this order in full volume at once. Otherwise when the order quantity exceeds the stock quantity ($Q_i > q$) the customer will receive only a part of goods, and there is the situation of deficit of $Q_i - q$ units of products in the wholesaler's warehouse.

Ordering cost $C_0(Q_i)$ has two components: constant c_1, which includes cost of the order forming and constant part of expenses of order transportation, and variable component $c_2(Q_i)$, which depends on the order quantity Q_i, i.e. $C_0(Q_i) = c_1 + c_2(Q_i)$. We assume that for all customers the holding cost is proportional to quantity of goods in stock and holding time with coefficient of proportionality C_{H}; losses from deficit are proportional to quantity of deficit with coefficient of proportionality C_{SH}. For i-th customer the average total cost in inventory system during one cycle $E_i^{\text{cust}}(\overline{T}_i)$ is calculated by the following formula:

$$E_i^{\text{cust}}(\overline{T}_i) = C_0(Q_i) + E_{\text{H}}(\overline{T}_i) + E_{\text{SH}}(\overline{T}_i), \; i = 1, 2, ..., n,$$

where $\overline{T}_i$ is average cycle time; $E_{\text{H}}(\overline{T}_i)$ is average holding cost during one cycle; $E_{\text{SH}}(\overline{T}_i)$ is average shortage cost during one cycle, and total cost E_i^{cust} per time unit for i-th customer can be found as divided by average cycle time $\overline{T}_i$ (Ross 1992):

$$E_i^{\text{cust}} = \frac{E_i^{\text{cust}}(\overline{T}_i)}{\overline{T}_i}, \quad i = 1, 2, ..., n. \qquad (3)$$

Note that $E_{\text{H}}(\overline{T}_i)$ and $E_{\text{SH}}(\overline{T}_i)$ depend on control parameters R_i and Q_i. Analytical formulas for these economical characteristics are presented in the paper (Kopytov and Greenglaz 2004). For problem solving we have to minimize criteria (3) by R_i and Q_i.

Second stage of ordering process

We assume that producer supplies its production to wholesaler according to a fixed schedule. In this case we consider ordering process with constant period of time T between the moments of placing neighbouring wholesaler's orders; and order quantity q is determined as difference between fixed stock level S and quantity of goods in the moment of ordering r (see Fig.3), i.e. $q = S - r$.

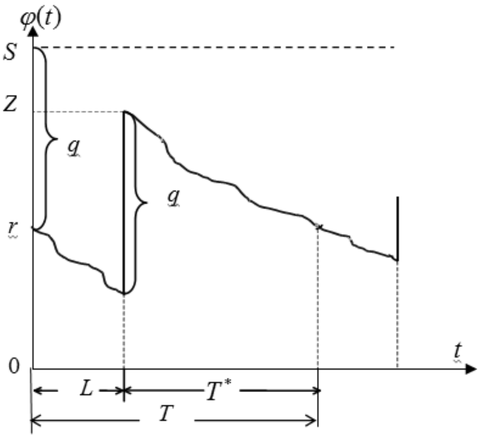

Figure 3: Dynamics of Wholesalers' Inventory Level During One Cycle

Let lead time L from the producer to the wholesaler have a normal distribution with a mean μ_L and a standard deviation σ_L. We suppose that lead time essentially less as time of the cycle: $\mu_L + 3\sigma_L << T$. We suppose that during time T the wholesaler has received orders from n customers, these orders can be described by the vector $\{Q_1, Q_2, ..., Q_n\}$. There is a possible situation of the deficit, when the demand $D(T) = \sum_{i=1}^{n} Q_i$ during time T exceeds the quantity of goods in stock Z in the time moment immediately after order receiving. Analogously to first stage we suppose that in case of deficit the last cannot be covered by expected order. We denote as S the goods quantity which is needed "ideally" for one period and it equals to the sum

$$S = \bar{D}(T) + S_0, \tag{4}$$

where $\bar{D}(T)$ is average demand during cycle time; S_0 is some safety stock (emergency stock).

We suppose that "ideally" S gives in future the minimum of total expenditure in inventory control system per unit of time. So, for the second stage in suggested model time period T and stock level S are control parameters. We suppose that in the moment of time, when a new order has to be placed, may be situation, when the stock level is so big that a new ordering doesn't occur. However for generality of model we'll keep the conception of lead time and quantity of goods at the time moment immediately after order receiving in such case too. It corresponds to real situation when the wholesaler uses the transport means, which depart at the fixed moments of time not depending on existence of the order and which have the random lead time; for example transportation by trailers which depart the 1st and 15th day of each month. In real situation in the moment of time t the stock level $\varphi(t)$ is equal to S only in two cases:

1) $r = S$ and $D(t) = 0$, where $D(t)$ is the demand for goods during the time t; $0 \le t \le T$;
2) $r < S$ and $D(t) = 0$, where $L \le t \le T$.

Taking into account that in case of deficit it can't be covered by expected order, we can obtain the expression for goods quantity at the moment of time immediately after order receiving:

$$Z = \begin{cases} r + q - D(L), & \text{if} \quad D(L) < r; \\ q, & \text{if} \quad D(L) \ge r, \end{cases}$$

where $D(L)$ is the demand during lead time L.

Using (4) we have:

$$Z = \begin{cases} S - D(L), & \text{if} \quad D(L) < r; \\ S - r, & \text{if} \quad D(L) \ge r. \end{cases}$$

Finally average total cost for time unit for the wholesaler is expressed by the following formula

$$E^{\mathrm{wh}} = \frac{E_{\mathrm{H}}^{\mathrm{wh}} + E_{\mathrm{SH}}^{\mathrm{wh}} + C_0(q)}{T}. \tag{5}$$

Unlike stage 1, in the considered stage expenditures $E_{\mathrm{H}}^{\mathrm{wh}}$ and $E_{\mathrm{SH}}^{\mathrm{wh}}$ depend on control parameters S and T.

SIMULATION MODEL IN EXTENDSIM 8 ENVIRONMENT

For the considered problem solving, the authors have used a simulation model realized in simulation package ExtendSim 8, which is the most powerful and flexible simulation tool for analyzing, designing, and operating complex systems in the market. It enables the researcher to test the hypotheses without having to carry them out. ExtendSim has repeatedly proven its being capable of modeling large complex systems (Krahl 2007).

Assume that in considered system we have three customers. The created simulation model for the supply chain "producer – wholesaler – customers" is shown in Fig. 4, 5, 7 and 8. The main screen of the simulation model is presented in Fig.4. Each zone of the model has a numeric label. In zone 1 an executive block, that controls all discreet events in Extend models, is placed. Zones 2 and 3 contain blocks which are responsible for modeling result representation: to a plotter block is placed in zone 2, and in zone 3 total expenses calculation and data export to Excel spreadsheet are executed. Zone 4 contains a block which is intended for scheduled transact generation; lead time and transport activity for goods transportation to the main store are simulated in blocks placed in zone 5. In the main storehouse zone there are placed: a block for holding activity simulation, a block for order quantity calculation, and an initialization block that performs queue initialization tasks before the model starts. In this situation all stocks are initialized before starting to represent a typical situation of goods quantity in the warehouse.

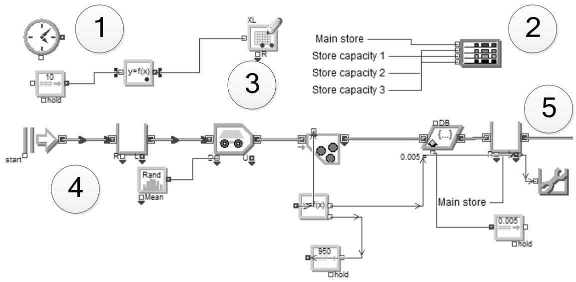

Figure 4: Main Screen of the Simulation Model

After goods delivery to the main warehouse they are transferred to customers' warehouses according to their orders. The hierarchical blocks shown in Fig.5 realize the "reorder point" strategy in goods ordering.

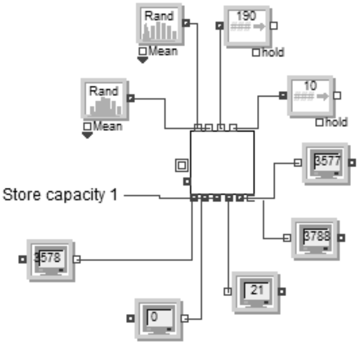

Figure 5: Reorder Point Store

This hierarchical block is made in the way, which allows using it in any necessary Extend model that needs such functionality. In the created model there are three identical reorder point blocks, for three customers stocks modeling respectively. For this reason, all control parameters and results are realized as input and output connectors. The internal parameters for this type of block are: stochastic lead time of goods delivery and demand for goods, shortage, delivery and holding costs, order quantity and reorder point. Specifying these parameters we can receive appropriate results, such as quantity of sold goods, amount of deficit, total costs that include ordering, holding and shortage costs. These result parameters are used for total cost calculation. Order quantity and reorder point are control parameters and have to be changed during the simulation procedure.

Using output connectors for goods quantity in stocks and plotter block, ExtendSim builds graphical representation of the dynamics of inventory level in all stocks shown in Fig.6.

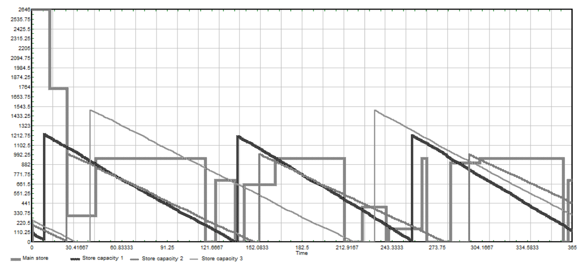

Figure 6: The Example of Simulation of the Inventory Control Process in All Stocks

Fig.7 illustrates internal structure of Reorder point hierarchical block. First block in zone 6 is called Gate, which allows or disallows transact entrance to this part of the model. Behavior of this block is controlled by Equation block that collects information about stock level, reorder point and placed order status. Based on the calculation of these parameters, Equation block sends Boolean value to Gate (0 – close and 1 – open). If transact is allowed for entrance, than it is passed to activity transport block (zone 7), after appropriate delay to the end store (zone 8). Blocks of zone 9 are used for expenses calculation. Zone 10 is used for internal communication between hierarchical block together

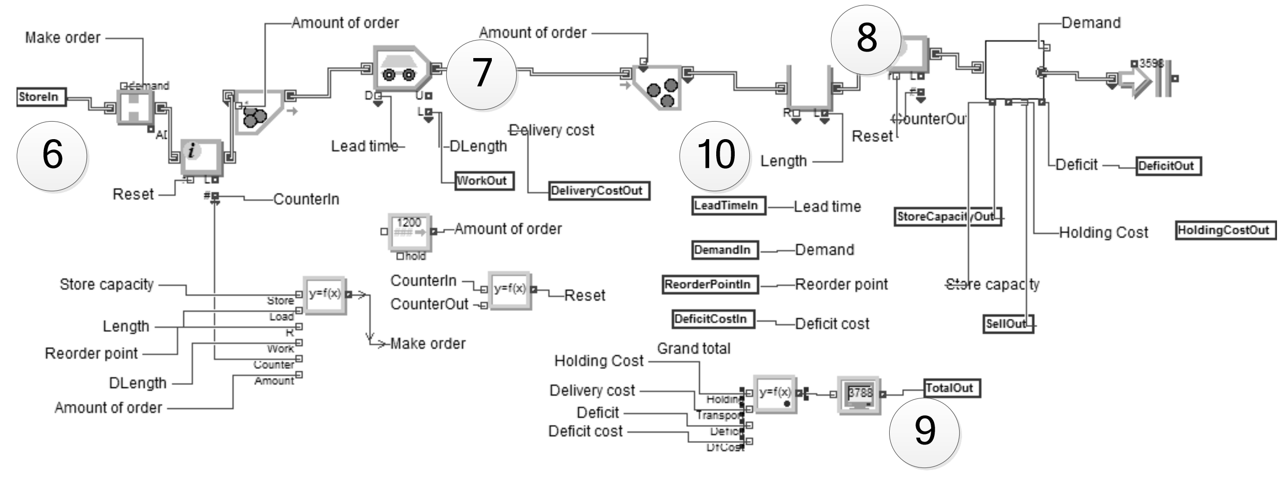

Figure 7: Reorder Point Hierarchical Block (internal)

with ExtendSim database.

Structure of the final hierarchical block is shown in Fig.8.

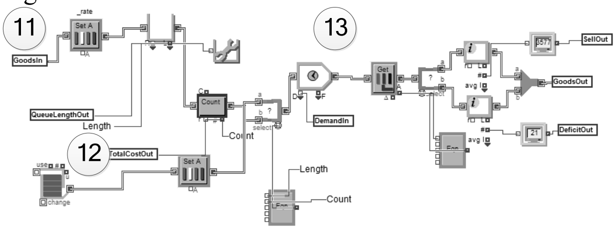

Figure 8: Hierarchical Block Customer Store

In zone 11 transactions are arriving to the warehouse, where they assigned the holding cost value. Zone 12 is designed for the deficit modeling with dummy supplier and appropriate attribute assigning. Zone 13 represents a market place where goods are sent to customers.

For end users' facilitation a specialized user interface was designed. Using this interface user can change control parameters of the model and get final results of simulation. There are several tools for user interface development in ExtendSim. One of them is Notebook window that can be called from any place of model and other is cloning tool that allows clone core control elements from ExtendSim block and place it into Notebook. The example of Notebook's window with initial data and results is shown in Fig.9.

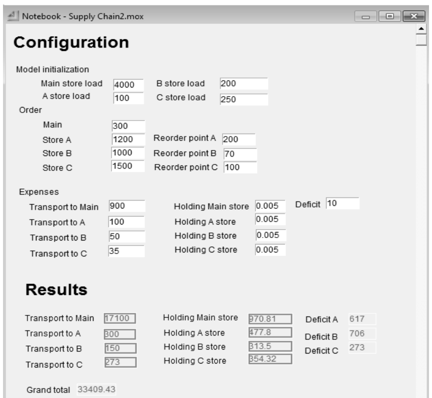

Figure 9: Example of Notebook's Window

EXAMPLE

There is considered a two-level inventory control system, which includes correspondingly a wholesale warehouse and the warehouses of 3 customers. For delivering the product there has been organized a supply chain "producer – wholesaler – customer" with two stages in the ordering process. It is assumed that all the customers are financially independent and organize the whole policy of ordering and holding the product by themselves; the wholesaler also acts only with the account of the minimization of his costs, losses from the product deficit included.

It is required to find such values of the reorder points R_1, R_2, R_3 with the customers and such value of the desired product stock S with the wholesaler at which the sum of the total costs of the goods' ordering and holding and the losses from the deficit per a unit of time unit would be minimal. The customers' demands for goods $D_i, (i = 1, 2, 3)$ are Poisson processes with intensity λ_i, and time L_i of goods delivery from wholesaler to *i*-th customer has normal distribution with parameters μ_i and σ_i (see Table 1). Ordering costs C_0 (including expenses of order transportation) for each customer are presented in Table 1 too.

Table 1: Initial Data

Customer, i	Lead time, L_i, days	Demand, λ_i units/day	Order quantity, Q_i, units	Initial stock, Z, units	Ordering cost, C_0, EUR
1	μ_i =11; σ_i =3.5	10	1200	100	100
2	μ_i =12; σ_i =2	8	1000	200	50
3	μ_i =14; σ_i =3.7	8.57	1500	250	35

The producer supplies its production to wholesaler according fixed schedule, and time period T between the moments of placing neighbouring orders is constant and equals 20 days. The policy of order forming for *i*-th customer is follows. A new order is placed in the moment of time, when the stock level falls to a certain level R_i .The time L of goods delivery from producer to wholesaler has a normal distribution with a mean $\mu_L = 3$ and a standard deviation $\sigma_L = 1$. Ordering cost C_0 (including expenses of order transportation) for wholesaler equals to 900 EUR. For customers and for wholesaler holding cost C_H equals to 0,005 EUR per unit per day, losses from deficit C_{SH} equal to 10 EUR per unit. Initial stock in wholesaler's warehouse is equal 4000 units. Fixed stock level in wholesaler's warehouse S is the control parameter of the model.

To solve the given task, we'll be using the simulation model described above and presented in Fig. 4, 5, 7 and 8. The period of simulation is one year and the number of replications is 100. There can be two strategies of optimization. The goal of the first strategy is summary minimization of all expenses for all model participants. The second strategy is optimization of individual customer and wholesaler activity. In this paper we'll look more closely at the first strategy.

After having performed the modeling of the initial variant of the system presented in Table 1, we'll get the following values of the control parameters: the reorder points with the customers, correspondingly, are

R1=200, R_2=70 and R_3=100 units; the level of the desired product stock with the wholesaler S=1350 units. And the value of the average total cost per year in the inventory system (see formula (1)) equals 22621 EUR. The given variant has been taken as the basic one.

Let us perform the optimization of the basic variant of the inventory control system. Note that due to limited volume of the given paper, we'll use only one control parameter from each pair: with the customer – it is the reorder point (the second parameter "order quantity" is fixed and determined in Table 1), with the wholesaler – it is the stock level (the interval between orders, as it has been noticed, equals to 20 days). With the account of the above assumptions about the economic independence of the particular customers and the wholesaler, it is suggested to use the algorithm of the step-by-step optimization. At each step of the proposed algorithm there is determined the value of the control parameter for the selected structural enterprise (first – customers, then – wholesaler), which, in the considered range of its change, gives the minimal value of the average total cost per year in the inventory system. Due to the illustrative character of the given article, the step of change of the control parameters with the customer is taken as 10 units, and with the wholesaler – 50 units. Let us consider the solution of the task in more detail.

Step 1

Using the data of the basic variant (see Table 1), let us perform the simulation of the stock system by changing the value of the reorder point, with Customer 1, in the range from 110 to 230 units with the step of 10, getting for each of the points 100 realizations. The results of the simulation are shown in Fig.10. Note that for the given steps of the control parameter R_1 changing the best result is achieved for reorder point R_1=190 units, where for 100 realizations average total cost E^{total} for one year period (see (1) equals 21838 EUR.

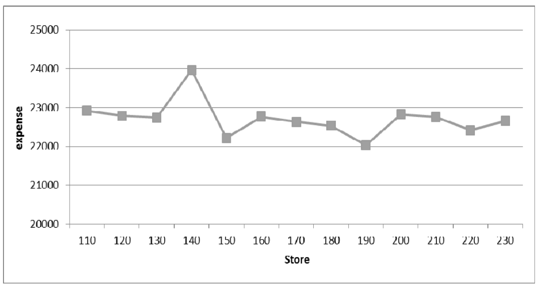

Figure 10: Average Total Expenses per Year in Inventory Control System (Step 1)

Step 2

Using the data received at step 1, let us perform the simulation of the stock system, changing the value of the reorder point, with Customer 2, in the range from 20 to 130 units. The results of the simulation are shown in Fig.11. For the given steps of the control parameter R_2 changing the best result is achieved for reorder point R_2=100 units, where average total cost E^{total} for one year period equals 21 813 EUR.

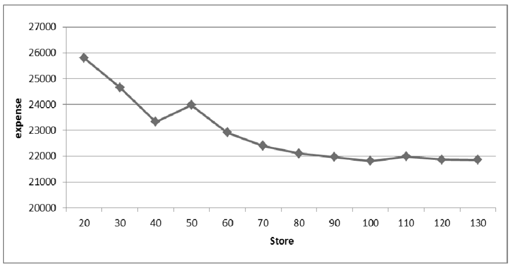

Figure 11: Average Total Expenses per Year in Inventory Control System (Step 2)

Step 3

Using the data received at step 2, let us perform the simulation of the stock system, changing the value of the reorder point, with Customer 3, in the range from 5 to 140 units. The results of the simulation are shown in Fig.12. For the given steps of the control parameter R_3 changing the best result is achieved for reorder point R_3=100 units, where average total cost for one year period E^{total} equals 21635 EUR.

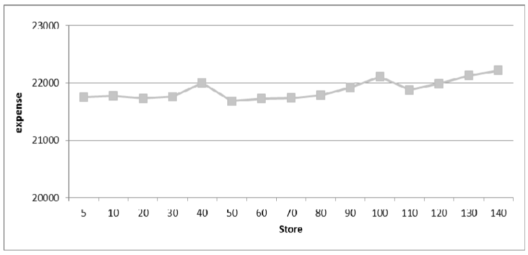

Figure 12: Average Total Expenses per Year in Inventory Control System (Step 3)

Step 4

Using the data received at step 3, let us change the level of the desired stock S with the wholesaler in the range from 900 to 1450 units and perform the simulation for different S values. The results of the simulation are shown in Fig.13. Note that for the given steps of the control parameter S changing the best result is achieved for reorder point S=1000 units, where for 100 realizations average total cost for one year E^{total} period equals 21527 EUR (see (1) and (5)).

The results of the considered steps are presented in Table 2. Note, that the optimal values of parameters received after each step are underlined. We cannot but notice that due to the change of values for the selected control parameters, we have managed to receive the best variant from the considered ones which provides the reduction of total cost E^{total} in the inventory control system, as compared with the source variant, by 1094 EUR or by 4,84%. It's clear that the given value cannot be seen as the minimal one since we have changed only one from the each pare of control parameters and used quite a large step of the parameters' changing.

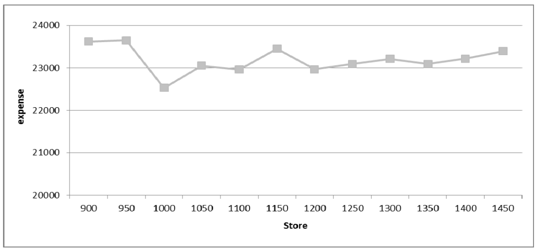

Figure 13: Average Total Expenses per Year in Inventory Control System (Step 4)

Table 2: The results of optimization process

Parameters	Values of control parameters				
	Base	Optimization steps in the model			
		Step 1	Step 2	Step 3	Step 4
Reorder point R_1, units	200	190	190	190	190
Reorder point R_2, units	70	70	100	100	100
Reorder point R_3, units	100	100	100	50	50
Desired stock level *S*, units	1350	1350	1350	1350	1000
Total expanses E^{total}, EUR	22621	21838	21813	21635	**21527**

CONCLUSIONS

The given paper has shown the possibility of using the ExtendSim 8 package for the simulation of a two-level inventory control system for the homogenous product stocks with the wholesaler's and customers' warehouses, characterized by a random demand for the product and random time of product delivery. The main advantages of the considered simulation method of inventory control problems solving are as follows:

- the clarity of the presentation of results; firstly, it touches the case of analysis of total expenses dependence on one control parameter with fixing others;
- the possibility of finding optimum solution of an inventory problem in the case when realization of analytical model is rather difficult;
- the descriptive user interface, and ability to control any necessary parameter.

The developed model can be used for examining the dynamics of the stocks' level at the warehouses of the customers and the wholesaler and for searching an optimal decision for the company having its own wholesale warehouse and a network of its own enterprises-customers. The future plan is investigation of different variants of wholesaler's ordering policy. Further guidelines of the current research are the following: to investigate multi-product inventory control problem with limitation on the existing resources (warehouse volume and monetary means).

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