METHOD OF EXPLICIT PREDICTIVE CONTROL IN CASE OF TIME-DELAYED HEAT SYSTEM

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ABSTRACT

This paper studies simulations of a possible appliance of an explicit form of the model predictive control method. Targeted system is a heat exchanger burdened with a time-delay. An approach is formulated within the MATLAB/SIMULINK development environment as well as the resulting simulation. The conclusion evaluates resulting system behaviour with focus on conditions in areas of practice benefiting from its use.

INTRODUCTION

Efforts to maximize the control precision of the industrial processes lead to an increased use of advanced control methods. The ability to optimize the control under the given circumstances and work with prediction of system behaviour has brought a significant attention towards the Model Predictive Control (MPC) methods. With efforts to implement control algorithms into a maximum number of systems appeared a requirement to decrease high computing demands necessarily connected with optimization procedures (Qin & Badgwell, 2003), (Xi et al., 2013). In order to allow an expansion to fast processes a number of modifications was presented. A radical course was set with an explicit form of the MPC (EMPC) presented by (Bemporad et al., 2000). The essential principle was to solve the optimization task outside of the control process.

This was extended in (Bemporad et al., 2002) for case of linear systems with constraints leading to multiparametric linear programming where were also discussed some issues connected with certain optimization methods. The explicit approach has been applied to control electrical drives (Linder & Kennel, 2005) experimentally proving applicability of this method. (Mariéthoz & Morari, 2008) proposed and experimentally tested a control strategy for a PWM inverter with attention to a switched behaviour of a converter. Results provided a good dynamic performance. Furthermore (Bolognani et al., 2011) tested controlling a synchronous motor drive with attention to possible modifications improving the performance.

In the effort to enable a faster control (Johansen & Granacharova, 2003) developed a search tree based real-time computation algorithm decreasing the computational complexity with a guarantee of stability and achievement of constraints. As with many functional simplifications, even in the case of EMPC a loss of certain functions occurred. The main reason lies in the fact that the extern optimization can compute with a limited amount of parameters. This prevents the involvement of a change in a desired trajectory, a measurable disturbance and so on. To overcome limitations of each individual approach (Zeilinger et al., 2008) proposed a combination of implicit and explicit methods aiming to satisfy needs of performance and computation time. As a result a hybrid control method was developed claiming suitability for most of general cases of systems. Despite getting increased attention due to its suitability to control fast processes, the necessary simplifications caused losing of several important aspect of on-line computed predictive control. Considering a potential loss of quality by using the explicit form due to its missing attributes a question arises whether its benefits still outnumber attached disadvantages.

This paper demonstrates differences between the implicit and the explicit form of the MPC control in case of a time-delayed system. Additionally a comparison of aspects aims to discuss preferable types of systems to control in the explicit way. The first section describes an example of both implicit and explicit control approaches intended to control a system with time-delay. The next segment contains simulated outcomes of a control process. The last part studies conditions determining a preferable version of the predictive control.

MODEL PREDICTIVE CONTROL

The principle of the model based predictive control lies in using an internal model which estimates a controlled system future development based on a current control input. The control approach is computed by an optimization task nearing a system output to a desired value.

With the predicted outputs involved in the computations this technique is a suitable way of controlling systems with time-delay as the predicted output replaces the control feedback. Therefore the controller is able to compute with outputs before they have even happened. The resulting quality of the control process highly de-

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pends on the precision of the internal model. Another benefit of this approach is a possibility to include a future development of the desired trajectory and therefore optimize a transition of the output value (Normey-Rico & Camacho, 2007).

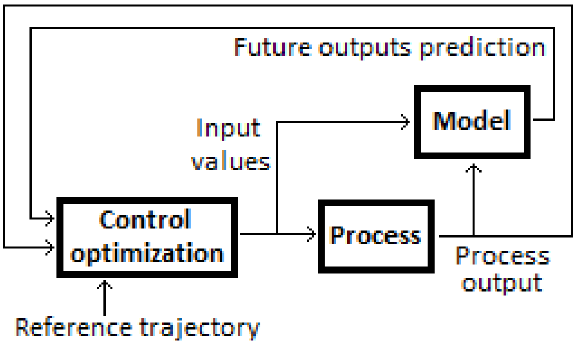

Figure 1: Structure of the Model Predictive Control

The optimization task itself is realized by minimizing an objective function containing the essential parameters of the control process. These parameters are so mathematically formulated that minimization of the objective function results in the best possible outcome. The general form of the objective function includes a difference between the estimated output values and the future desired trajectory on one hand, on the other differences in the future control input. The balance between these parts is determined by weighting values and their ratio. The general expression of the objective function is

$$J=\sum_{i=N_1}^{N_2}\delta(i)\left[\hat{y}(k+i)-w(k+i)\right]^2+\sum_{i=1}^{N_u}\lambda(i)\left[\Delta u(k+i-1)\right]^2, \quad (1)$$

where $\delta(i)$ and $\lambda(i)$ are weighting values.
The number of estimated steps and naturally the complexity of calculation are determined by horizons N_1 and N_2 called minimal and maximal horizon while N_u as a control horizon limits the calculation of control input changes (Camacho & Bordons, 2004).
The optimization task produces a series of control input values from which only the first one is applied into the control process. In the next sampling step the whole procedure is repeated based on new updated information about the system states. Due to involvement of the future desired trajectory development the system is able to optimize the control for changes that are about to happen; therefore the process does not have to wait for the time of change in the desired value and it can start adjusting several sampling steps ahead. This enables a fluent transition during sudden changes in the desired trajectory. The concept of applying the first control value and repeating the optimization at each sampling step is called a receding horizon strategy which is illustrated in Figure 2.

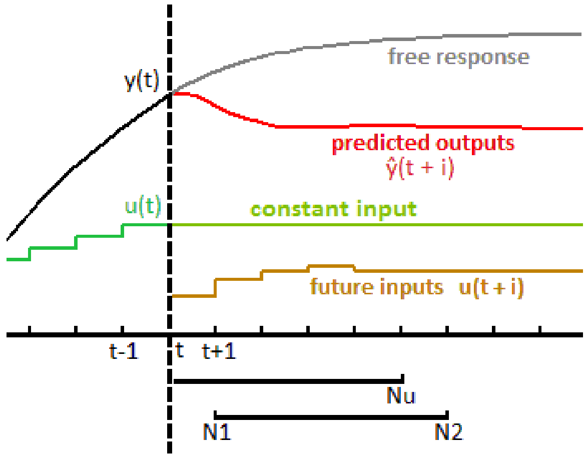

Figure 2: Receding Horizon Strategy

The estimation of the future output values is based on a combination of free and forced response with the superposition principle. The free response **f** is given by a prediction with a constant input. The forced response calculated from a series of control input values and system parameters.

$$\hat{\mathbf{y}}=\mathbf{G}\mathbf{u}+\mathbf{f}\,. \quad (2)$$

The optimal series of control inputs is then gained from searching for the minimal value of the objective function (1) while the output estimation is received from (2). The following section describes a predictive control approach based on state-space internal model (Haber et al., 2011).

State-space model based predictive control

The implementation of the state-space internal model into the predictive control requires a state observer which increases the computation demands. Nonetheless, benefits of this approach are an easier application to multi-input multi-output (MIMO) systems and a possibility to include conditions regarding system states into the optimization.
In order to apply this internal model it needs to be transformed from the traditional structure into an incremental form, which enables using the aforementioned objective function (1).
While the generic discrete state-space system is given as

$$\begin{aligned}x(k+1)&=\mathbf{A}x(k)+\mathbf{B}u(k)\\ y(k)&=\mathbf{C}x(k)+\mathbf{D}u(k)\end{aligned} \quad (3)$$

then with the definition of the control input increment

$$\Delta u(k)=u(k)-u(k-1) \quad (4)$$

is the new version written as

$$\begin{aligned}x(k+1)&=\mathbf{A}x(k)+\mathbf{B}\Delta u(k)+\mathbf{B}u(k-1)\\ y(k)&=\mathbf{C}x(k)\end{aligned} \quad (5)$$

while the effect of matrix **D** is neglected.
By extending the vector of states by a previous control input an incremental structure is received

$$\begin{bmatrix} x(k+1) \\ u(k) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ 0 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{I} \end{bmatrix} \cdot \Delta u(k) \qquad (6)$$

$$y(k) = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}$$

This can be rewritten into a shorter version

$$\begin{aligned} \widetilde{x}(k+1) &= \widetilde{\mathbf{A}} \cdot \widetilde{x}(k) + \widetilde{\mathbf{B}} \cdot \Delta u(k) \\ y(k) &= \widetilde{\mathbf{C}} \cdot \widetilde{x}(k) \end{aligned} \qquad (7)$$

The future outputs estimation is derived from new versions of the system matrices while the equation (2) is computed using matrices

$$\mathbf{G} = \begin{bmatrix} \widetilde{\mathbf{C}}\widetilde{\mathbf{B}} & 0 & \cdots & 0 \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{A}}\widetilde{\mathbf{B}} & \widetilde{\mathbf{C}}\widetilde{\mathbf{B}} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{A}}^N\widetilde{\mathbf{B}} & \cdots & \cdots & \widetilde{\mathbf{C}}\widetilde{\mathbf{B}} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \widetilde{\mathbf{C}}\widetilde{\mathbf{A}} \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{A}}^2 \\ \vdots \\ \widetilde{\mathbf{C}}\widetilde{\mathbf{A}}^N \end{bmatrix} \qquad (8)$$

when the free response is received from

$$\mathbf{f} = \mathbf{F}\widetilde{x}(k) \qquad (9)$$

resulting in series of output values for case of constant control input. Here should be noted that in case of time-delayed system the content of the matrix **F** is shifted by a number of delayed sampling steps d ahead.

The series of control inputs is then calculated by finding the minimum of the objective function (1). The search for the extreme could be simplified to a derivation of the task by the control input while neglecting constrains in the following equation

$$\mathbf{u} = -(\mathbf{G}^T\mathbf{Q}\mathbf{G} + \mathbf{R})^{-1}\mathbf{G}^T\mathbf{Q}(\mathbf{f} - \mathbf{w})$$

Where matrices **Q** and **R** represent weighting parameters (Shields et al., 2006).

Explicit form of predictive control

In order to decrease the significant computation demands that are present in the above described implicit version of predictive control, a transformation of the computing process has been made. The struggle aims to transfer the complex computations from each sampling step of the control process out into the preparatory phase. Mainly the optimization task as the most demanding area needs to be solved before the start of the control process. As state of the process can obviously vary during the control phase, the optimization needs to be performed for as much cases as possible. The result is a function depending on the current state of the process giving the key element for the control input computation (Bemporad et al., 2002).

In order to keep the number of computations in the preparatory phase reasonably small and the complexity of control phase low, an amount of simplifying variations was presented. As a reasonable approximation designated areas restricted to intervals of state values were created. Determining the proper region can have fixed rules or be based on a specialized algorithm. In each of these areas the control input computation is based on constant coefficients, therefore the final computation of the input requires only basic relation between the state and the corresponding constant.

Before the start of the control process the optimization task of the objective function (1) is performed for determined range of values of system states creating a function dependant on states as coordinates. With sufficient amount of calculations an estimation of the function can be made. The whole function shape is split into regions determined either by an approximating algorithm further increasing computation demands, or fixed borders often unable to capture essential shapes of the function. The control phase in ideal case needs only current values of process states emphasizing the need of state identifier. Received values represent coordinates determining region of approximation where the controller is currently located. Based on the parameters gained in the starting phase from the optimization task the control input value for the current step is calculated.

In order to regulate the output value not towards zero, but in the direction of the desired trajectory, there needs to be involved a difference between the estimated and the desired output. The control law is therefore directed to minimize the control deviation instead of the pure value of the state. A procedure of including the time-delay compensation into the control law requires a calculation of a system free response for a sufficient number of steps into the future. This method applies an **F** matrix from equations (8) and (9), where similarly to the implicit control approach the focus is towards the value in position $d + 1$ estimating the output after the delay. Furthermore, with the knowledge of the desired trajectory and by including its future value instead of the current one, the negative effect of the delay can be partially eliminated. Due to the computation with only a single value of output deviation given by the principle of EMPC, the controller is unable to prepare for a future change in the desired trajectory without still remaining near to its present value.

EXPERIMENTAL LABORATORY HEAT EQUIPMENT

A scheme of the laboratory heat equipment (Pekař et al., 2009) is described in Figure 3. The heat transferring fluid (e. g. water) is transported using a continuously controllable DC pump (F) into a flow heater (A) with max. power of 750 W. The temperature of a fluid at the heater output T_1 is measured by a platinum thermometer. Warmed liquid then goes through a 15 meters long insulated coiled pipeline (B) which causes the significant delay (20 – 200 s) in the system. The air-water heat exchanger (C) with two cooling fans (D, E) represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers as T_2, respective T_3. The platinum thermometer T_4 is dedicated for measurement of the outdoor-air temperature. The laboratory heat equipment is connected to a standard PC via technological multifunction I/O card MF 624. This card is designed for the need of connecting PC compatible computers to real world signals. The card is designed for standard data acquisition, control applications and optimized for use with Real Time Toolbox for SIMULINK. The MATLAB/SIMULINK environment was used for all monitoring and control functions.

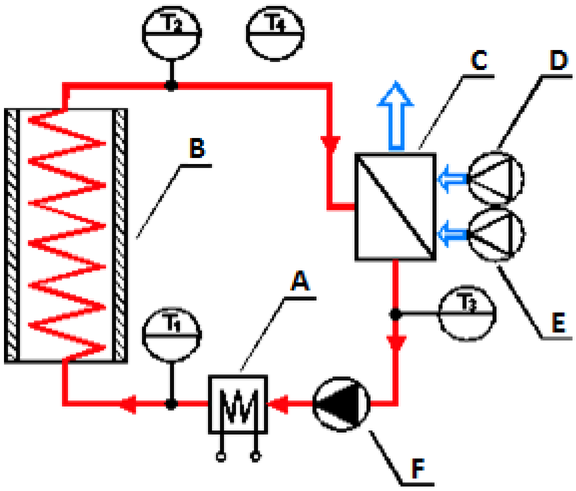

Figure 3: Scheme of Laboratory Heat Equipment

CONTROL PROCESS SIMULATION

In the following section will be described simulations of applying the control algorithms to a simple stable non-oscillatory system first and then the above mentioned heat exchanger.
Values of horizons specifying the predictive control computations were set based on the system step response to hold most of the function. Values of the maximal and control horizons were in both cases established to 50 sampling steps and for the heat system the minimal horizon was moved to 2 due to the time-delay effect. Weighting values were both set to 1 to equally distribute optimization calculations.
System states were periodically estimated by a state observer as a part of the control process.
A shape of the desired trajectory was designed to demonstrate various difficulties of a regulation. The first stage has a shape of step change up followed by a step change down. The next part has similar form but with ramp shaped changes providing a constant increase of the desired trajectory. The trajectory ends with a stage of constant value to stabilize the output.

Testing system

System parameters were selected to provide a simple stable process with a significant time-delay of 5 sampling steps in order to demonstrate variations in use of implicit and explicit approaches.
The system transfer function is

$$G(z) = \frac{0.4728z^{-1} - 0.2076z^{-2}}{1 - 0.7419z^{-1} - 0.0821z^{-2}} z^{-5}$$

with sampling step 2 seconds long.

In state-space expression

$$A = \begin{bmatrix} -0.0217 & -0.1571 \\ 0.6283 & 0.7636 \end{bmatrix}, \quad B = \begin{bmatrix} 0.6283 \\ 0.9456 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.5 \end{bmatrix}, \quad D = 0$$

Regulation results gained by using implicit and explicit control methods can be seen on following figures.

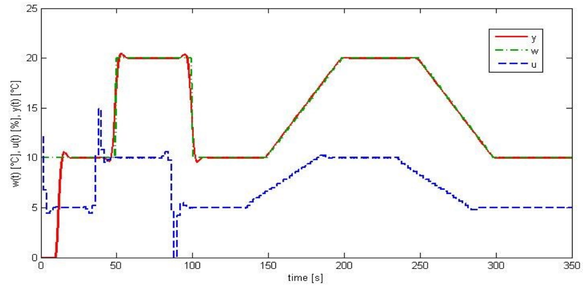

Figure 4: Implicit MPC Process with Time-Delay

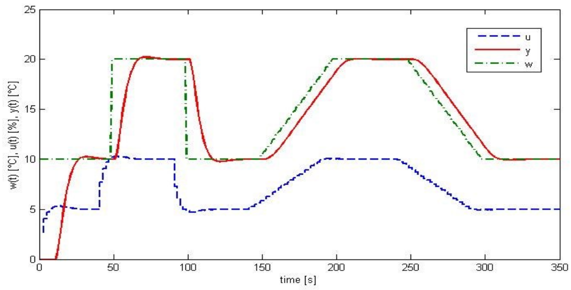

Figure 5: Explicit MPC Process with Time-Delay

Figures 4 and 5 reveal some small differences in the control procedure. The most obvious one is the response to the desired trajectory where in case of the explicit control the reaction is delayed up to the time of the current change in the desired value. This is caused by its inability to compute with the further future changes of the desired trajectory as it would immensely increase computation demands. Therefore the controller is unable to perform a smooth preparation for the step change unlike the implicit control method. Furthermore, in the stage of ramp shaped desired trajectory is a noticeable gap creating an extended control error. The cause of this pattern originates in the estimation of the constant desired value as the controller can operate only with one. Nevertheless, this particular effect could be negated by providing the algorithm a desired trajectory shifted more towards the future, specifically in the area of ramps.

Heat exchanger

The laboratory heat model was identified with the recursive least square method giving the discrete result as a stable second order non-oscillatory system described by the following transfer function

$$G(z) = \frac{0.1619z^{-1} - 0.0957z^{-2}}{1 - 0.7948z^{-1} - 0.0754z^{-2}} z^{-2}$$

with a sampling period of 60 seconds.
In state-space expression

$$A = \begin{bmatrix} 0.1398 & -0.0011 \\ 27.0833 & 0.9569 \end{bmatrix}, B = \begin{bmatrix} 27.1 \\ 1058.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0016 & 0 \end{bmatrix}, D = 0$$

The duration of the time-delay with the current setting was measured to be approximately 120 seconds which is fitting to two sampling intervals.

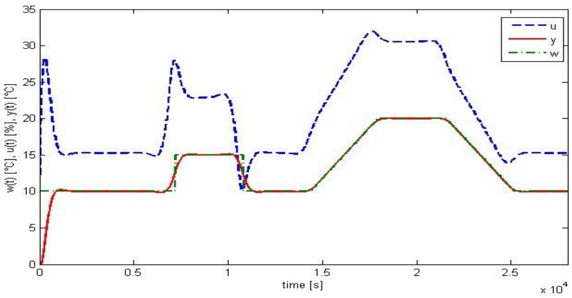

Figure 6: Implicit MPC Process of the Heat System

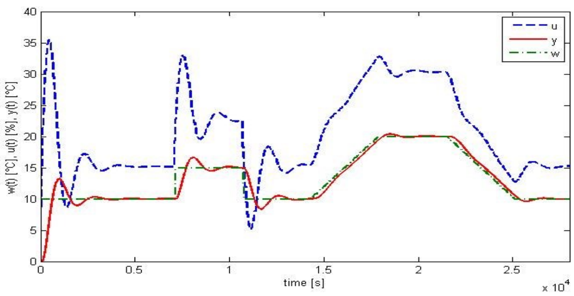

Figure 7: Explicit MPC Process of the Heat System

Results of simulated control process are illustrated in Figure 6 for an implicit MPC and on Figure 7 for an explicit method. The effect of the time-delay is decreased by computing with future values of the desired trajectory, consequently the controller reacts in the actual moment of the required change.
Aside from the differences mentioned in the evaluation of the previous simulation the explicit version displays overshoots after step changes of the desired trajectory. Unlike the implicit optimizing approach, the explicit form is unable to compute with the whole past and future system dynamic. This would require too high computing power which would be in conflict with the original purpose of the explicit strategy. As an easier solution, the end of a transition between distant desired levels can be smoothed by modifying weighting parameters of the optimization task to prioritize the control precision.
In order to numerically evaluate the quality of both control method ISE criterion was selected

$$ISE = \int_0^\infty [e(t)]^2 \, dt$$

This criterion was applied on simulations of the heat model visualized in Figures 6 and 7.

Table 1: ISE Criterion Comparison

Method	ISE criterion value
Implicit	$3{,}35 \cdot 10^4$
Explicit	$5{,}52 \cdot 10^4$

As can be seen in Table 1 the implicit version provides a better quality due to above mentioned properties.
Considering the computation complexity of both methods it is safe to assume the significantly easier on-line computation of the explicit version as it replaces the optimization task with a few multiplication operations. To verify this times of computations occurring in simulations were measured.

Table 2: Times of Computations

Method	Preparatory	Control - 467 steps	1 step
Implicit	-	136,07s	0,2914s
Explicit	60,20s	1,77s	0,0038s

Table 2 describes how much time was required to perform computations for each method. The implicit approach calculates optimal constrained control series directly during the control phase consisting of 467 sampling steps. The explicit one is divided to the preparatory phase creating necessary optimizations and the control phase. Demands of the preparatory computations creating variations of regions dependent on system states highly rely on the applied optimization and mapping algorithms therefore offering a vast range of possibilities to decrease this particular time.

CONCLUSION

The paper compares an application of two strategies of a predictive control method to a laboratory heat system burdened with time-delay. Outcomes of simulations prove both methods to be feasible solutions for a time-delayed system.
However differences in control processes suggest significantly contrasting areas of application.
A much lower computing complexity of the explicit method during the control process offer an approach to control fast processes with lower needs of computing hardware.
The presence of time-delay is successfully compensated in both strategies and brings an equal vulnerability to unexpected effects for each of them. The case of large number of delay steps tends to be more disadvantageous for the explicit form as can increase computing demands above expected amounts.
Without the knowledge of the future desired value development the process loses a significant amount of precision which is especially disadvantageous in a time-delayed system control. The use of implicit optimizing strategy would then lose a great benefit of optimizing the process with attention to the future development and consequently degrade the overall quality to computation demand ratio.
The explicit approach, however, does not enable expansion of the control process with an adaptation function

as it would require repeating the whole initial computations. This repetition would dramatically increase computational complexity during the regulation making the explicit principle contra productive and ultimately pointless. Therefore the use of the explicit form is limited to cases estimated during the algorithm design.
The future research work will focus on expanding the precision of the explicit form and its flexibility under external conditions.

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