SIMULATION OF A QUEUEING MODEL USEFUL IN CROWDSOURCING

Srinivas R. Chakravarthy
Department of Industrial and
Manufacturing Engineering
Kettering University
MI 48504, Flint, USA
E-Mail: schakrav@kettering.edu

Serife Ozkar
Department of Statistics
Istanbul Medeniyet University
Istanbul, Turkey
E-mail: serife.ozkar@medeniyet.edu.tr

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ABSTRACT

In this paper we study a multi-server queueing model in the context of crowdsourcing useful in service sectors. There are two types of arrivals to the system such that one group of customers after getting service from the system may serve (with a certain probability) another group. Through simulation we point out, even for a small value of this probability, the significant advantage in considering crowdsourcing by offering more traffic load (through increasing the rate of online customers without violating the stability condition) to the system resulting in more customers served (which in turn increasing the revenues when cost/profits are incorporated). Also, the role of correlation, especially a positive one, present in the inter-arrival times in the system performance measures is highlighted.

INTRODUCTION

The concept of crowdsourcing has been used in different domains gaining significant exposure in many service sectors. We refer the reader to a recent survey paper by (Hosseini et al. 2015 and Evans et al. 2016). The meaning and interpretation of crowdsourcing is varied and despite its popularity among companies in many sectors it remains little understood. We refer the reader to (Howe 2008) for a number of examples related to crowdsourcing in various sectors.

The literature on the quantitative analysis of the crowdsourcing with the help of mathematical models is very small even though the literature on qualitative nature of crowdsourcing dealing with various definitions and classification is huge. The quantitative models for crowdsourcing will benefit business and service industries to better understand the system when underlying parameters change. For example, with the help of survival analysis and using the dataset from MTurk, (Wang et al. 2011) analyzed the completion time of crowdsourcing campaigns. The material flow of crowdsourcing processes in manufacturing systems was studied by using stochastic Petri nets in (Wu et al. 2014). Only recently stochastic models, more specifically queueing models, useful in crowdsourcing in the context of service sectors have been studied. To the best of our knowledge the first queueing model using one type of customers as possible servers for another group was studied by (Chakravarthy and Dudin 2017). The authors studied a queueing with corowdsourcing of *M/M/c*–type using matrix-analytic methods.

It should be pointed out that the model studied in (Bernstein et al. 2012) deals with retaining a select few workers as "backup" servers on call for helping the system. These workers are allowed to tend to other tasks until a request for their help is made by the system. Thus, in their model the customers arriving at the system are never considered as servers for the system.

In this paper, we generalize the model studied in (Chakravarthy and Dudin 2017) by considering a more versatile point process, namely, Markovian arrival process (*MAP*) for Type 1 arrivals, Poisson arrivals for Type 2, and phase type (*PH*-distribution) for services. That is, we study queueing model of *MAP/PH/c*–type with crowdsourcing and resort to simulation for the analysis since the state space of the queueing model grows exponentially with the number of servers, phase of the arrival, and the phase of the service processes.

It is well-known (Neuts 1975) that a PH-distribution is obtained as the time until absorption in a finite-state Markov chain with an absorbing state. Realizing the limitations of Poisson processes and exponential distributions in spite of their nice mathematical properties, Neuts (Neuts 1979) first developed the theory of phase type distributions and *MAPs*. The *MAP* is a rich class of point processes that not only generalize many well-known processes such as Poisson, *PH*-renewal processes, and Markov-modulated Poisson process but also provides a way to model *correlated arrivals.* For further details on *MAP* and their usefulness in stochastic modelling, we refer to (Lucantoni et al. 1990; Lucantoni 1991; Neuts 1992) and for a review and recent work on *MAP* we refer the reader to (Artalejo et al. 2010; Chakravarthy 2001; Chakravarthy 2010).

MODEL DESCRIPTION

We consider a c- server queueing system in which two types, say, Type 1 and Type 2, of customers arrive. We assume that Type 1 customers arrive according to a MAP with representation (D_0, D_1) of order m. An arriving Type 1 customer finding the server idle will get into service immediately. Otherwise, the customer will enter a finite

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buffer of size L, $1 \leq L < \infty$, to be served on a First-Come-First-Served (*FCFS*) basis when the server becomes free. Thus, it is possible for a Type 1 customer to be lost at the time of arrival due to the buffer being full. Let D, defined by $D = D_0 + D_1$, govern the underlying Markov chain of the *MAP* such that D_0 accounts for the transitions corresponding to no arrival; D_1 governs those corresponding to an arrival of a Type 1 customer. By assuming D_0 to be a nonsingular matrix, the interarrival times will be finite with probability one and the arrival process does not terminate. Hence, we see that D_0 is a stable matrix. Let λ_1 denote the arrival rate of Type 1 customers. The arrivals of Type 2 customers are assumed to follow a Poisson process with rate λ_2. There is no restriction on how many Type 2 customers can be in the system. That is, there is an infinite buffer space for Type 2 customers.

While Type 1 customers are to be served by one of the *c* servers, Type 2 customers may be served by a Type 1 customer having already been served and also available to act as a server or by one of the *c* (system) servers. For example, Type 1 customers visit the store to buy items while Type 2 customers order over some medium such as Internet and phone, and expects them to be delivered. The store management can use the in-store customers as couriers to "serve" the other type of customers. Not all in-store customers may be willing and in some cases not possible to act as servers for the store. Hence, a probability is introduced for Type 1 customers to opt for servicing Type 2 customers.

A Type 2 customer getting serviced by a Type 1 customer depends on the following conditions. First, that Type 1 customer should have just finished getting a service and opts to service a Type 2 customer. Secondly, at the time of opting to serve there is at least one Type 2 customer waiting to get a service. We assume that a served Type 1 customer will be available to act as a server for a Type 2 customer under the conditions mentioned above with probability p, $0 \leq p \leq 1$. With probability $q = 1 - p$, the served Type 1 customer will leave the system without opting to serve a Type 2 customer. Upon completion of a service a free server will offer service to a Type 1 customer on a *FCFS* basis; however, if there are no Type 1 customers waiting, the server will serve a Type 2 customer if there is one present in the queue. If a Type 1 customer decides to serve a Type 2 customer, for our analysis purposes that Type 2 customer will be removed from the system immediately. This is due to the fact that the system no longer needs to track that Type 2 customer.

We assume that all system servers offer services to either type on a *FCFS* within the type; however, Type 1 customers have non-preemptive priority over Type 2 customers. The service times are assumed to be of phase type with representation $(\boldsymbol{\beta}, S)$ of order *n* with mean $1/\mu = \boldsymbol{\beta}(-S)^{-1}\boldsymbol{e}$, where $\boldsymbol{e}$ is a column vector of 1's of order *n* here and will be of dimension of appropriate dimension in the sequel.

The arrival rate, λ_1, is given by $\lambda_1 = \boldsymbol{\delta} D_1 \boldsymbol{e}$, where $\boldsymbol{\delta}$ is the stationary probability vector of the irreducible generator D, and is the unique (positive) probability vector satisfying $\boldsymbol{\delta} D = \boldsymbol{0}, \boldsymbol{\delta e} = 1.$

The model outlined above can be studied as a Markov process by keeping track of (a) the number, $K_1(t)$, of Type 2 customers in the queue; (b) the number, $K_2(t)$, Type 1 customers in the queue; (c) the number, $K_3(t)$, of servers busy serving Type 1 customers; (d) the number, $K_4(t)$, of servers busy with Type 2 customers; (e) the phase , $J_r(t)$, of the r^{th} server, and (f) the phase, $J(t)$, of the arrival process at time *t*. The process $\{(K_1(t), K_2(t), K_3(t), K_4(t), J_1(t), \cdots, J_{K_3(t)+K_4(t)}(t), J(t): t \geq 0)\}$ is a continuous-time Markov chain with state space given by

$$\Omega = \{(i_1, i_2, i_3, i_4, j_1, \cdots, j_c, k): i_1 \geq 0,\ 0 \leq i_2 \leq L,\ 0 \leq i_3 \leq c,\ 0 \leq i_4 \leq c,\ 0 \leq i_3 + i_4 \leq c,\ 0 \leq j_r \leq n\ 1 \leq r \leq c, 1 \leq k \leq m\}.$$

Note that we take $J_r = 0$ when the r^{th} server is idle. The generator of this Markov process can be set up with the help of Kronecker products and sums of matrices. However, it is clear that the analysis of this model analytically requires a large state space to account for all the states described above. These are currently work-in-process and the results will be reported elsewhere. However, our goal in this paper is to see how the impact of introducing crowdsourcing in the context of multi-server queueing system with *MAP* arrivals through simulation. The rest of the paper is based on simulating the crowdsourcing queueing model described here with the help of ARENA (Kelton et al. 2010). Unless otherwise mentioned, we ran our simulation models using 3 replications and for 1,000,000 units (which in our case is minutes) for each replicate.

VALIDATION OF THE SIMULATED MODEL

In any simulation work, it is important to validate the simulated model by comparing the results with any known analytical results. Hence, we will do that in this section. The only cases for which analytical results are available for the model under study are for *M/M/c* (Chakravarthy and Dudin 2017). This is due to recent interests to study crowdsourcing from queueing theory perspective.

We will list four key system performance measures among many for our illustration.

- The probability, *PLOS*, that an arriving Type 1 customer is lost due to the buffer being full.
- The probability, *PT*2*L*1, that a Type 2 customer leaves (served) with a Type 1 customer.
- The mean, *MN*1*Q*, number of Type 2 customers waiting time in the queue.
- The mean, *MN*2*Q*, number of Type 2 customers waiting time in the queue.

M/M/c crowdsourcing

In (Chakravarthy and Dudin 2017), the authors studied an *M/M/c* type queueing models with crowdsourcing. We will compare our simulated results against their numerical ones, which were generated through analytical study by employing matrix-analytic methods. Specifically, we fix $\lambda_1 = 1$, $\mu = 1.1, L = 10$, and vary other parameters as follows: $p = 0, 0.5, 1$, $\rho = 0.8, 0.99$, $c = 1, 2, 5, 10$. The values of λ_2, which depend on the (fixed) value of ρ (see (Chakravarthy and Dudin 2017)), are displayed in Table 1 below.

Table 1: Values of λ_2 for various scenarios for *M/M/c*

	$\rho = 0.8$			$\rho = 0.99$		
c	$p = 0$	$p = 0.5$	$p = 1$	$p = 0$	$p = 0.5$	$p = 1$
1	0.1232	0.5016	0.8800	0.1524	0.6207	1.0890
2	0.9602	1.3601	1.7600	1.1882	1.6831	2.1780
5	3.6000	4.0000	4.4000	4.4550	4.9500	5.4450
10	8.0000	8.4000	8.8000	9.9000	10.3950	10.8900

The error percentage, which is calculated as $\{|Analytical - simulated| \ / \ Analytical\}$ 100%, for various scenarios are displayed in Table 2.

By looking at Table 2 we notice that our simulated results are very close to the ones obtained using analytical results presented in (Chakravarthy and Dudin 2017) for all except a couple of scenarios. For these scenarios we ran the simulation again but with 10,000,000 minutes and 3 replicates and found the error percentages for these cases drop significantly.

Table 2: Error percentages (%) of analytical and simulated models for *M/M/c*

		MN1Q		*MN2Q*		*PLOS*		*PT2L1*	
c	p \ ρ	0.8	0.99	0.8	0.99	0.8	0.99	0.8	0.99
1	0	0.13	0.20	0.66	7.24	0.00	0.00	0.00	0.00
	0.5	0.42	0.22	1.04	11.98	0.39	0.93	0.11	0.13
	1	0.30	0.16	0.10	22.47	1.16	0.00	0.10	0.07
2	0	0.10	0.05	0.51	7.05	0.00	0.00	0.00	0.00
	0.5	0.17	0.07	0.88	8.68	0.00	0.00	0.00	0.03
	1	0.09	0.00	0.07	3.05	0.00	0.00	0.02	0.02
5	0	0.22	0.18	0.28	0.80	0.00	0.00	0.00	0.00
	0.5	0.07	0.00	0.43	7.02	0.00	0.00	0.12	0.10
	1	0.07	0.05	0.20	2.77	0.00	0.00	0.07	0.11
10	0	0.22	0.10	0.19	1.16	0.00	0.00	0.00	0.00
	0.5	0.00	0.00	0.12	0.59	0.00	0.00	0.00	0.21
	1	0.20	0.00	0.26	1.00	0.00	0.00	0.35	0.00

SIMULATED RESULTS FOR *MAP/PH/c* CROWDSOURCING

For the arrival process, we consider the following five sets of values for D_0 and D_1 as follows.

Erlang distribution (*ERLA*):

$$D_0 = \begin{pmatrix} -2 & 2 \\ 0 & -2 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

The exponential distribution (*EXPA*):

$$D_0 = (-1), D_1 = (1)$$

The hyper-exponential distribution (*HEXA*):

$$D_0 = \begin{pmatrix} -1.90 & 0 \\ 0 & -0.19 \end{pmatrix}, \ D_1 = \begin{pmatrix} 1.710 & 0.190 \\ 0.171 & 0.019 \end{pmatrix}$$

The MAP with negative correlation (*MNCA*):

$$D_0 = \begin{pmatrix} -1.00222 & 1.00222 & 0 \\ 0 & -1.00222 & 0 \\ 0 & 0 & -225.75 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0.01002 & 0 & 0.9922 \\ 223.4925 & 0 & 2.2575 \end{pmatrix}$$

The MAP with positive correlation (*MPCA*):

$$D_0 = \begin{pmatrix} -1.00222 & 1.00222 & 0 \\ 0 & -1.00222 & 0 \\ 0 & 0 & -225.75 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0.9922 & 0 & 0.01002 \\ 2.2575 & 0 & 223.4925 \end{pmatrix}$$

These *MAP* processes will be normalized to have a specific arrival rate. However, these are qualitatively different in that they have different variance and correlation structure. The first three arrival processes, namely, *ERLA*, *EXPA*, and *HEXA*, have zero correlation for two successive inter-arrival times. The arrival processes labeled *MNCA* and *MPCA*, respectively, have negative and positive correlation for two successive inter-arrival times with values -0.4889 and 0.4889. The ratio of the standard deviation of the inter-arrival times of these five arrival processes with respect to *ERLA* are, respectively, 1, 1.41421, 3.17450, 1.99335, and 1.99335.

For the service times $(\boldsymbol{\beta}, S)$ we consider the following three PH-distributions.

The Erlang distribution (*ERLS*):

$$\beta = (1 \quad 0), S = \begin{pmatrix} -2 & 2 \\ 0 & -2 \end{pmatrix}$$

The exponential distribution (*EXPS*):

$$\beta = (1), S = (-1)$$

The hyper-exponential distribution (*HEXS*):

$$\beta = (0.9 \quad 0.1), S = \begin{pmatrix} -1.90 & 0 \\ 0 & -0.19 \end{pmatrix}$$

Notice that these three *PH*-distributions all have the same mean but are qualitatively different in that the variations in the distributions are different. The ratio of the standard deviation of these three service times with respect to

ERLS are, respectively, 1, 1.41422, and 3.17454. These distributions will be appropriately normalized to attain a specific mean in the numerical examples.

Having validated our simulated crowdsourcing model for known cases in the previous section, we will now present a few illustrative examples to bring out qualitative nature of *MAP/PH/c* crowdsourcing model under study. We will discuss three examples here. In the sequel, we let Y denote the waiting time in the system of a Type 2 customer. In addition to the measures listed in the validation section we consider the following ones.

- The probability, *PIDL*, the system is idle.
- The probability, *PBUS*1, that the system is busy serving Type 1 customers.
- The probability, *PBUS*2, that the system is busy serving Type 2 customers.
- The mean, $MWTS_1$, waiting time in the system of Type 2 customer.
- The mean, $MWTS_2$, waiting time in the system of Type 2 customer.
- The fraction, *FATH*, of Type 2 customers whose waiting time in the system exceeds $r,\ r \geq 2$, times the average service time by one of the system servers. That is, $P\left(Y > \frac{r}{\mu}\right), r \geq 2.$ Since there is no analytical expression available for the measure, *FATH*, dealing with a specific tail probability of the waiting time in the system of a Type 2 customer, we used the simulated result instead. It should be pointed out that one can compute algorithmically the tail probability for classical single-server model, *MAP/PH/1* using the matrix-analytic methods (Neuts 1981) but for a multi-server case it is highly complicated and hence we resort to simulation only for this particular measure.

The purpose of our next example is to investigate the level of such effect *MAP/PH/c* case using simulated results. We will look at the ratio of a few of the measures under study here. The ratio, $\frac{\eta(p>0)}{\eta(p=0)}$, will be of interest for a given measure η.

EXAMPLE 1: The effect of crowdsourcing is studied in this example by comparing the models: (a) $p = 0$ that corresponds to having two independent arrival processes and (b) $p > 0$ that corresponds to the crowdsourcing model in the context of a multi-server system. In the latter case there is a possibility for Type 1 customers to act as servers for Type 2. We fix $\lambda_1 = 1, \lambda_2 = 2, \mu = 1.1, c = 3, L = 10$ and vary p on the interval $(0, 1]$ under different combinations of arrival and service distributions.

In Figure 1 below we display the ratios of all but *PBUS*1 since the ratio for *PBUS*1 for all scenarios are almost 1. Some key observations are summarized as follows. First observe that the smaller the ratio the better the system in terms of all measures except *PIDL* in which case it should be the larger the better. Having more idle time for the system will enable the management to use that time for other activities without having to lower the quality of service provided to the customers.

In Figure 1, we display the ratios of the key measures under different scenarios for *ERLS* and *HEXS* services. Note that for lack of space we display only selected combinations; however, our observations summarized below are valid for other combinations not displayed in this figure.

A quick look at these figures reveals the following interesting and important observations.

– The ratio for the measure, *PIDL*, is greater than 1 and increases as p is increased indicating that the server becomes idle more often when $p > 0$ when compared to that of $p = 0$. This is true for all combinations of arrivals and services.

– The ratios of all other measures are less than 1 and decrease as p is increased. The rate of decrease depends on the type of arrivals and services.

– The ratio for $MWTS_2$ decreases significantly as p increases. This is true for all cases. This is very important from both management's as well as customers' points of view.

– The ratio for $MWTS_1$ decreases as p increases but not as significantly as that of $MWTS_2$. This is somewhat surprising since one would expect the ratio to be close to 1 since Type 1 service is not affected by the value of p due to non-preemptive nature of services. However, as p increases, Type 1 customers have a higher probability of serving Type 2 customers resulting in relatively fewer Type 2 customers to be served by one of the system servers and hence a reduction in the mean waiting time in the system.

– The ratio for *FATH* is decreasing at a significantly higher rate as p increases (for all scenarios) which is again very important from both management as well as customers' points of view and can also be used by the management to guarantee some kind of a guarantee on the service times of Type 2 customers.

The above observations show the significant advantage in introducing this type of variants, namely, crowdsourcing, to the classical queueing models.

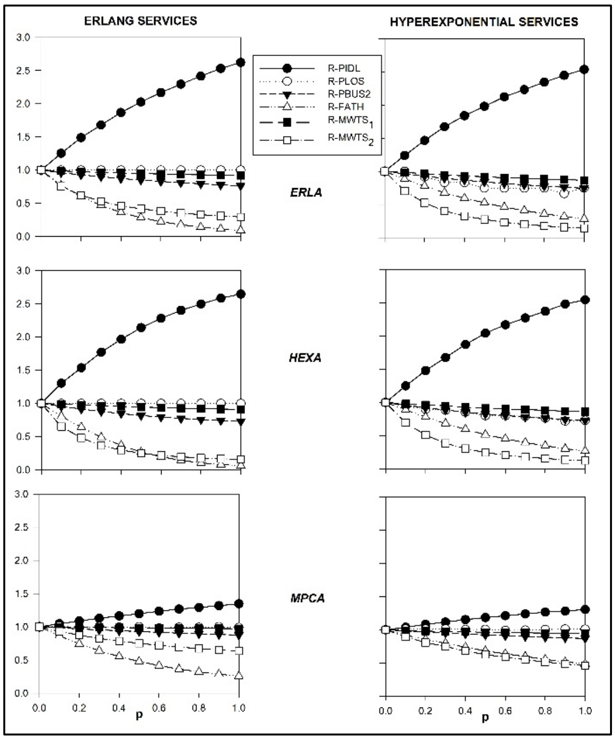

Figure 1: Ratios of various measures under different scenarios with *ERLS* and *HEXS* services

In (Chakravarthy and Dudin 2017), it was shown that even in the case of small p there is a significant advantage in considering crowdsourcing by offering more traffic load through increasing the rate of Type 2 customers into the system. The rate of increase in the offered load to the system is much higher for small values of c. Here, we will investigate a similar advantage from a different point of view by considering the cases when $L = 0$ and $L = 1$. In the former case Type 1 customers are allowed only when at least one server is idle. Thus, the maximum number of Type 1 customers that can be present at any time in the system is c and $c + 1$, respectively.

EXAMPLE 2: This example is very similar to Example 1 except that we look at the cases when (a) $L = 0$ and (b) $L = 1$. Note that in these two cases the model can be considered as a slight variation of *M/PH/c* model since Type 2 arrivals arrive to a multi-server system with phase type arrivals and occasionally Type 1 customers are allowed to enter into the system. Hence, it will be interesting to see how having only a small number of Type 1 customers, namely, c and $c + 1$ when $L = 0$ and $L = 1$, respectively, at any given time will have an impact on the selected system performance measures. Towards this end, we will fix $\lambda_1 = 1$, $\lambda_2 = 2$, $\mu = 1.1$, $c = 3$, and vary p on the interval $(0,1]$ under different combinations of arrival and service distributions. Note that the queue is stable for all combinations under these values. In order to properly compare, we now look at the ratio $\frac{\zeta(L=1)}{\zeta(L=0)}$ where $\zeta(L = r) = \frac{\eta(p>0,L=r)}{\eta(p=0,L=r)}$, $r = 0, 1$.

In Figure 2 below we display the ratios for selected measures and for representative scenarios. First observe that the smaller the ratio the better the system with $L = 1$ as compared to $L = 0$ in terms of all measures except *PIDL* in which case it should be the larger the better. Having more idle time for the system will enable the management to use that time for other activities without having to lower the quality of service provided to the customers.

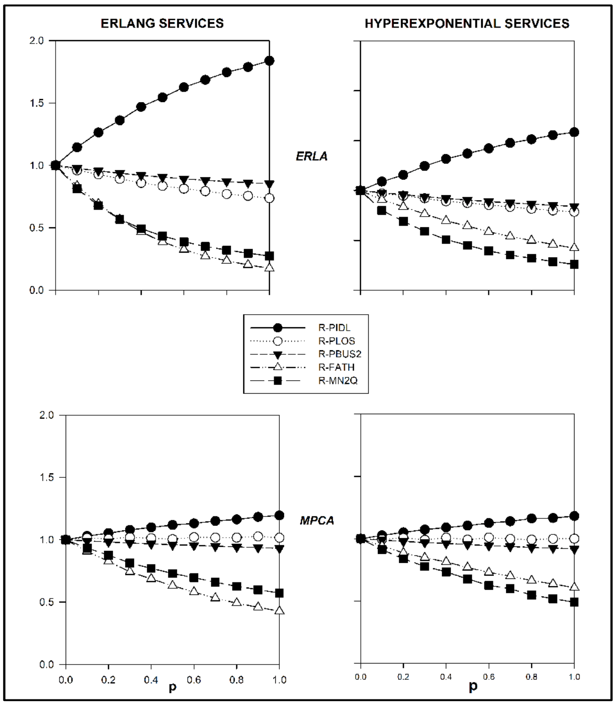

Figure 2: Comparison of the ratio under different scenarios with *ERLS* and *HEXS* services

A quick look at this Figure reveals a very significant advantage of having extra Type 1 customers in the system. This is the case for all scenarios (even the ones not displayed here for lack of space) and for all values of p. So, having Type 1 customers even if they are willing to offer services rarely plays a significant role in crowdsourcing applications.

In the next example, we try to find the optimum c^* such that the proportion of Type 2 customers whose waiting time in the system exceeds r, $r \geq 2$, times the average service time by one of the system servers does not exceed a pre-determined value, such as 5%. Recalling that Y denotes the waiting time in the system of a Type 2 customer, then for any $c \geq c^*$ when all other parameters are fixed, the following holds good.

$$P\left(Y > \frac{r}{\mu}\right) < 0.05, r \geq 2.$$

The purpose of this is to identify the regions where for a given p the minimum value of c that will guarantee that only certain (pre-determined) percentage of Type 2 customers has longer than a (pre-determined) multiple of the average service time. Similarly, we can fix c and identify if there is any p that will yield a similar guarantee.

EXAMPLE 3: Here we fix $\lambda_1 = 1$, $\mu = 1.1$, $L = 10$, and vary other parameters as follows: $\rho = 0.8, 0.9$, $p = 0, 0.5, 1$.

In Table 3 we display the optimum c^* for various combinations. We ran our simulation for c up to 50 and if an optimum is not found in that range, we will denote this by simply displaying with "> 50".

Table 3: Optimum c^* values

		ERLS			*EXPS*			*HEXS*		
ρ	*MAP*	*p*=0	*p*=0.5	*p*=1	*p*=0	*p*=0.5	*p*=1	*p*=0	*p*=0.5	*p*=1
0.8	*ERLA*	3	2	2	4	4	3	>50	>50	>50
	EXPA	3	2	2	5	4	3	>50	>50	>50
	HEXA	3	3	2	5	4	3	>50	>50	>50
	MNCA	3	2	2	5	4	3	>50	>50	>50
	MPCA	3	2	2	20	4	3	>50	>50	>50
0.99	*ERLA*	3	3	2	4	4	3	>50	>50	>50
	EXPA	3	3	2	10	4	3	>50	>50	>50
	HEXA	3	3	2	5	4	3	>50	>50	>50
	MNCA	3	3	2	4	4	3	>50	>50	>50
	MPCA	3	3	2	20	4	3	>50	>50	>50

A quick look at this table suggests that for *HEXS*, which has a large variation compared to the other two service distributions, one needs c to be larger than 50 for all values of p. Only in the case of positively correlated arrivals and with non-Erlang service times we see a relatively large c when Type 1 customers are not willing to serve Type 2 customers (i.e., when $p = 0$) which is not surprising as positively correlated arrivals are known to show such "odd" behavior with regard to other system performance measures in the literature (see e.g., (Chakravarthy 2010)).

CONCLUSION

In this paper we considered a queueing system useful in crowdsourcing. Specifically, we considered a multi-server queueing model in which one type of customers may be available to serve another type of customers leading to more efficiency as well as to help the management to increase their productivity and hence revenues. Through illustrative numerical examples obtained via simulation, we showed the significant benefits in introducing this type of variants to the classical queueing models.

Even for the small value of the probability introduced for in-store customers to opt for servicing the other type, we point out the significant advantage in considering crowdsourcing by offering more traffic load (through increased the rate of online customers without violating the stability condition) to the system resulting in more customers served (which in turn increasing the revenues when cost/profits are incorporated). Also, the role of correlation, especially a positive one, present in the inter-arrival times in the system performance measures is highlighted.

The model considered in this paper can be improved as follows. The assumption that Type 2 customers may be served singly by Type 1 customers can be relaxed to include batch services.

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AUTHOR BIOGRAPHIES

SRINIVAS R. CHAKRAVARTHY is professor of Industrial Engineering and Statistics in the Departments of Industrial and Manufacturing Engineering & Mathematics at Kettering University (formerly known as GMI Engineering & Management Institute), Flint, Michigan. His research interests are in the areas of algorithmic probability, queuing, reliability, inventory, and simulation. He has published more than 100 papers in leading journals and made more than 90 presentations at national and international conferences. He co-organized the First International Conference on MAMs in Stochastic Models in 1995 held in Flint. His recognitions and awards include Rodes Professor, Kettering University, Kettering University Distinguished Research Award, Kettering University/GMI Alumni Outstanding Teaching Award, GMI Outstanding Research Award, and GMI Alumni Outstanding Teaching Award, and Educator of the Year Award by IEOM Society, 2016. Srinivas Chakravarthy has significant industrial experience by consulting with GM, FORD, PCE, and UPS. His professional activities include serving as (a) Area Editor for the journal, Simulation Modelling Theory and Practice; (b) Associate Editor for the journal IAPQR TRANSACTIONS-Indian Association for Productivity, Quality & Reliability; (c) Advisory Board Member for several other journals and International Conferences; and (d) Reviewer for many professional journals.

SERIFE OZKAR is a research assistant pursuing her doctoral studies in the Department of Statistics, Istanbul Medeniyet University. This paper was written while she was a visiting research scholar in the Department of Industrial and Manufacturing Engineering, Kettering University, Flint, Michigan. Serife Ozkar's research interests are in the areas of queuing systems and stochastic processes.