# PREDICTIVE CONTROL OF TWO-INPUT TWO-OUTPUT SYSTEM WITH NON-MINIMUM PHASE

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#### **KEYWORDS**

Simulation, Model Predictive Control, Linear Systems, Non-Minimum Phase Systems, TITO Systems.

### ABSTRACT

In this paper, a simulation of predictive control of a two-input two-output (TITO) system with nonminimum phase is presented. The proposed controller is based on extended setting of constraints. This setting represents a modification for purposes of control of nonminimum phase multivariable systems. The main problem of control of this particular type of system is undesirable undershoot in the initial phase of the control. Known methods can properly reduce the undershoot in case of predictive control of single-input single-output (SISO) systems. The paper proposes a modification of a predictive controller for two-input two-output systems with non-minimum phase behaviour. The non-minimum phase TITO models are simulated using its mathematical representation in the form of transfer function matrix.

### **INTRODUCTION**

Control of systems which are characterized by nonminimum-phase behavior (Hoagg and Bernstein, 2007) requires a quite sophisticated approach for a controller design. The main problem is an undershoot which is present during the control using classical controllers without any modifications taking into account the nonminimum phase behaviour of the controlled system. A suitable method for control of the non-minimum-phase systems is model predictive control (MPC) (Huang, 2002; Rawlings and Mayne, 2009; Corriou, 2004). A successful implementation of MPC for control of nonminimum phase SISO systems is presented for example in (Barot and Kubalcik, 2014). Generally, many technological processes require a simultaneous control of several variables related to one system. In this case, a design of a controller is more sophisticated. One of the most effective approaches to control of multivariable systems is model predictive control. An advantage of model predictive control is that multivariable systems can be handled in a straightforward manner. Therefore, model predictive control appears as a suitable approach for control of non-minimum phase multivariable systems.

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Model Predictive Control is one of the control methods which have developed considerably over a few past years. Predictive control is essentially based on discrete or sampled models of processes. Computation of appropriate control algorithms is then realized especially in the discrete domain. The basic idea of the generalized predictive control (Camacho and Bordons, 2007; Kwon, 2005) is to use a model of a controlled process to predict a number of future outputs of the process. A trajectory of future manipulated variables is given by solving an optimization problem incorporating a suitable cost function and constraints. Only the first element of the obtained control sequence is applied. The whole procedure is repeated in the following sampling period. This principle is known as the receding horizon strategy.

The known approach for control of non-minimum-phase SISO systems (Camacho and Bordons, 2007) consists of increasing of a minimum control horizon. However, this classical modification may not be generally successful. The further possibility is setting of equality constraints applied in several initial sampling periods of control (Barot and Kubalcik, 2014). This modification was also tested for control of non-minimum phase TITO systems. The obtained results were not satisfactory. The undershoots were not eliminated using this approach.

In this paper, an appropriate setting of constraints in MPC which eliminates undershoots in the initial phase of model predictive control of non-minimum phase multivariable systems is presented. This approach was implemented for control of a TITO non-minimum phase system.

## MODEL OF THE CONTROLLED TITO SYSTEM

A continuous TITO system can be expressed using the transfer function matrix:

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$
(1)

A non-minimum phase behaviour is characterized by positive values of roots  $\mathcal{G}_i; i \in \langle 1; 4 \rangle$  in numerators in partial transfer functions  $G_{ij}; i, j \in \langle 1; 2 \rangle$  of the transfer function matrix (2).  $\pi_l; l \in \langle 1; 8 \rangle$  are poles.

$$\boldsymbol{G}(s) = \begin{bmatrix} \frac{s - \theta_1}{(s - \pi_1)(s - \pi_2)} & \frac{s - \theta_2}{(s - \pi_3)(s - \pi_4)} \\ \frac{s - \theta_3}{(s - \pi_5)(s - \pi_6)} & \frac{s - \theta_4}{(s - \pi_7)(s - \pi_8)} \end{bmatrix}$$
(2)

In the discrete simulation of MPC, the model (2) is expressed in Z-transform (Kučera, 1991) for a given sampling period T as (3).

$$\boldsymbol{G}(z^{-1}) = \begin{bmatrix} G_{11}(z^{-1}) & G_{12}(z^{-1}) \\ G_{21}(z^{-1}) & G_{22}(z^{-1}) \end{bmatrix}$$
(3)

For the simulation purposes, the mathematical model of TITO system (3) can have a form of the matrix fraction (4).

$$\boldsymbol{G}(z^{-1}) = \boldsymbol{A}^{-1}(z^{-1})\boldsymbol{B}(z^{-1})$$
(4)

The structure of the particular matrices  $A(z^{-1})$  and  $B(z^{-1})$  can be seen in (5)-(6) with description of polynomials in (7)-(8).

$$A(z^{-1}) = \begin{bmatrix} \alpha_{11}(z^{-1}) & \alpha_{12}(z^{-1}) \\ \alpha_{21}(z^{-1}) & \alpha_{22}(z^{-1}) \end{bmatrix}$$
(5)

$$\boldsymbol{B}(z^{-1}) = \begin{bmatrix} \beta_{11}(z^{-1}) & \beta_{12}(z^{-1}) \\ \beta_{21}(z^{-1}) & \beta_{22}(z^{-1}) \end{bmatrix}$$
(6)

$$\alpha_{11}(z^{-1}) = 1 + \alpha_{111}z^{-1} + \alpha_{112}z^{-2};$$
  

$$\alpha_{12}(z^{-1}) = \alpha_{121}z^{-1} + \alpha_{122}z^{-2};$$
  

$$\alpha_{21}(z^{-1}) = \alpha_{211}z^{-1} + \alpha_{212}z^{-2};$$
  

$$\alpha_{22}(z^{-1}) = 1 + \alpha_{221}z^{-1} + \alpha_{222}z^{-2}$$
(7)

$$\beta_{11}(z^{-1}) = \beta_{111}z^{-1} + \beta_{112}z^{-2};$$
  

$$\beta_{12}(z^{-1}) = \beta_{121}z^{-1} + \beta_{122}z^{-2};$$
  

$$\beta_{21}(z^{-1}) = \beta_{211}z^{-1} + \beta_{212}z^{-2};$$
  

$$\beta_{22}(z^{-1}) = \beta_{221}z^{-1} + \beta_{222}z^{-2}$$
(8)

The transformation from the continuous model (1) to its discrete version (4) is possible using the least squares method (Nelles, 2001). The parameters of the polynomials (7)-(8) were included in an ARX model (Nelles, 2001) of the TITO system.

# MODEL PREDICTIVE CONTROL OF TITO SYSTEMS

The model predictive control is a control strategy which incorporates a model of the controlled system for predictions of output variables. The calculations are performed on the receding horizon window which corresponds to a maximum prediction horizon  $N_2$ . The control action signal is denoted as u(k). The output signal is y(k), e(k) is a control error and w(k) is a reference signal. Each variable in TITO MPC is twodimensional.

The vector of future increments of manipulated variable  $\Delta u$  with  $N_u$  elements is determined by solving an optimization task which comprises a suitable cost function and constraints of variables.  $N_u$  is a control horizon. In the optimization task, the unknown variable y is determined by prediction equations (11) where the future outputs of the controlled model are determined by CARIMA model (Controlled AutoRegressive Integrated Moving Average) (Rossiter, 2003) (9). Equation (9) can be rewritten to equations (10) and (11) without consideration of the noise signal  $e_s(k)$ .  $N_1$  and  $N_2$  are minimum and maximum prediction horizons. Matrices P and g are defined in (12)-(14) where Z is a zero matrix of a given dimension.

$$A(z^{-1})\mathbf{y}(k) = \mathbf{B}(z^{-1})\mathbf{u}(k) + \Delta^{-1}(z^{-1})\mathbf{C}(z^{-1})\mathbf{e}_{s}(k)$$
  
$$\Delta(z^{-1}) = \begin{bmatrix} 1 - z^{-1} & 0 \\ 0 & 1 - z^{-1} \end{bmatrix}$$
(9)

$$\mathbf{y}(k) = \mathbf{A}_{1} \mathbf{y}(k-1) + \mathbf{A}_{2} \mathbf{y}(k-2) + \mathbf{A}_{3} \mathbf{y}(k-3) + \\ + \mathbf{B}_{1} \Delta \mathbf{u}(k-1) + \mathbf{B}_{2} \Delta \mathbf{u}(k-2); \\ \mathbf{A}_{1} = \begin{bmatrix} (1-\alpha_{111}) & -\alpha_{121} \\ -\alpha_{211} & (1-\alpha_{221}) \end{bmatrix}; \\ \mathbf{A}_{2} = \begin{bmatrix} (\alpha_{111} - \alpha_{112}) & (\alpha_{121} - \alpha_{122}) \\ (\alpha_{211} - \alpha_{212}) & (\alpha_{221} - \alpha_{222}) \end{bmatrix}; \\ \mathbf{A}_{3} = \begin{bmatrix} \alpha_{112} & \alpha_{122} \\ \alpha_{212} & \alpha_{222} \end{bmatrix}; \mathbf{B}_{1} = \begin{bmatrix} \beta_{111} & \beta_{121} \\ \beta_{211} & \beta_{221} \end{bmatrix}; \\ \mathbf{B}_{2} = \begin{bmatrix} \beta_{112} & \beta_{122} \\ \beta_{212} & \beta_{222} \end{bmatrix}$$
 (10)

$$\begin{bmatrix} \mathbf{y}(k+N_{1}) \\ \vdots \\ \mathbf{y}(k+N_{2}) \end{bmatrix} = \mathbf{P} \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{y}(k-1) \\ \mathbf{y}(k-2) \\ \Delta \mathbf{u}(k-1) \end{bmatrix} + \mathbf{G} \begin{bmatrix} \Delta \mathbf{u}(k) \\ \Delta \mathbf{u}(k+1) \\ \vdots \\ \Delta \mathbf{u}(k+N_{u}-1) \end{bmatrix}$$
(11)  
$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \cdots & \mathbf{P}_{14} \\ \mathbf{P}_{21} & \mathbf{P}_{22} & \cdots & \mathbf{P}_{24} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_{i1} & \mathbf{P}_{i2} & \cdots & \mathbf{P}_{i4} \end{bmatrix};$$
  
$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \cdots & \mathbf{G}_{1j} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \cdots & \mathbf{G}_{2j} \\ \vdots & \ddots & \vdots \\ \mathbf{G}_{i1} & \mathbf{G}_{i2} & \cdots & \mathbf{G}_{ij} \end{bmatrix}$$
(12)

$$\begin{array}{c}
\boldsymbol{\mathcal{G}} \in \mathcal{R}^{2N_{2} - 2N_{1} + 2, 2N_{2}};\\\\
\boldsymbol{\mathcal{G}} = \boldsymbol{Z}; \boldsymbol{Z} \in \mathcal{R}^{2N_{2} - 2N_{1} + 2, 2N_{2}};\\\\
\boldsymbol{\mathcal{G}}_{11} = \boldsymbol{\mathcal{G}}_{22} = \boldsymbol{\mathcal{G}}_{33} = \boldsymbol{B}_{1};\\\\
\boldsymbol{\mathcal{G}}_{21} = \boldsymbol{\mathcal{G}}_{32} = (\boldsymbol{A}_{1}\boldsymbol{B}_{1} + \boldsymbol{B}_{2});\\\\
\boldsymbol{\mathcal{G}}_{31} = (\boldsymbol{A}_{1}^{2}\boldsymbol{B}_{1} + \boldsymbol{A}_{1}\boldsymbol{B}_{2} + \boldsymbol{A}_{2}\boldsymbol{B}_{1});\\\\\\
\begin{pmatrix}\boldsymbol{\mathcal{G}}_{i1} = \boldsymbol{A}_{I}\boldsymbol{\mathcal{G}}_{(i-1)1} + \boldsymbol{A}_{2}\boldsymbol{\mathcal{G}}_{(i-2)1} + \boldsymbol{A}_{3}\boldsymbol{\mathcal{G}}_{(i-3)1}\\\\\\ \boldsymbol{\mathcal{G}}_{i(j-I)} = \boldsymbol{A}_{I}\boldsymbol{\mathcal{G}}_{(i-1)(j-1)} + \boldsymbol{A}_{2}\boldsymbol{\mathcal{G}}_{(i-2)(j-1)} + \\\\\\ + \boldsymbol{A}_{3}\boldsymbol{\mathcal{G}}_{(i-3)(j-1)} + \boldsymbol{B}_{2}\\\\\\ \boldsymbol{\mathcal{G}}_{ij} = \boldsymbol{B}_{I} \\\\\\ i = 4, \dots, N_{2}; \ j = 1, \dots, i \end{array}\right), \qquad (13)$$

$$P \in \mathcal{G}_{c} = \gamma,$$

$$P_{11} = A_{1}; P_{12} = A_{2}; P_{13} = A_{3}; P_{14} = B_{2};$$

$$P_{21} = A_{1}^{2} + A_{2}; P_{22} = A_{1}A_{2} + A_{3};$$

$$P_{23} = A_{1}A_{3}; P_{24} = A_{1}B_{2};$$

$$P_{31} = A_{1}^{3} + A_{1}A_{2} + A_{3} + A_{1}A_{2};$$

$$P_{32} = A_{1}^{2}A_{2} + A_{1}A_{3} + A_{2}^{2};$$

$$P_{33} = A_{1}^{2}A_{3} + A_{3}A_{2}; P_{34} = A_{1}^{2}B_{2} + A_{2}B_{2};$$

$$\left(P_{ij} = A_{1}P_{(i-1)j} + A_{2}P_{(i-2)j} + A_{3}P_{(i-3)j}\right),$$

$$i = 4, ..., N_{2}; j = 1, ..., i$$

$$(14)$$

The optimization problem is then solved as a minimization by the quadratic programming. A cost function J is defined by (15)-(16) where the vector  $\Delta u$  is solved with regard to the constraints defined by matrix inequality (17). Matrix I is an identity matrix.

$$M\Delta u \le \gamma \tag{17}$$

#### CLASSICAL MODIFICATIONS FOR MPC OF NON-MINIMUM PHASE SISO SYSTEMS

The method of predictive control enables a particular setting of the horizons for the purposes of control of non-minimum phase systems. The recommended approach increases the minimum horizon  $N_1$ . It means

reducing of  $N_1$  upper rows in matrices P and G for SISO control or  $2N_1$  rows for TITO control. Simulations performed for SISO systems proved that this method can not be generally successfully applied with appropriate results. The undesired undershoots were not eliminated in all cases.

Therefore, further possible modification was proposed in (Barot and Kubalcik, 2014) which successfully eliminated undershoots for SISO systems. The principal of this method consists of particular equality constraints settings (18) of  $\Delta u_{max}$  in the initial part of control. E.g., the constraint value was obtained experimentally as a relatively small constant  $\zeta$ . For TITO systems, the experiments were not successful using the approach which was successfully used for SISO systems.

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}; \boldsymbol{M} \in \mathcal{R}^{N_{u}, N_{u}}$$
(18)

$$\boldsymbol{\gamma} = \begin{bmatrix} \Delta u_{\max} \equiv \zeta \\ \vdots \\ \Delta u_{\max} \equiv \zeta \end{bmatrix}; \boldsymbol{\gamma} \in \mathcal{R}^{N_u, 1}$$
(19)

# MODIFICATION FOR MPC OF NON-MINIMUM PHASE TITO SYSTEMS

TITO MPC of non-minimum phase systems is more complicated than SISO MPC. The efforts to eliminate the undershoot using the methods described in the previous section were not successful. In this section it is introduced a modification which provided satisfactory control results without undesired undershoot also for TITO non-minimum phase systems.

The main principle consists of restriction of the controlled variable y. In this case, restricted is the lower limit of y in the initial part of control. This restriction is applied only in the initial part of control. The matrix inequality (17) is then modified to matrices (20) and (21).

$$\boldsymbol{M} = -\boldsymbol{\mathcal{G}}; \, \boldsymbol{M} \in \mathcal{R}^{2(N_2 - N_1 + 1), 2N_u}$$
(20)

$$\boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{y}(k) \\ \boldsymbol{y}(k-1) \\ \boldsymbol{y}(k-2) \\ \Delta \boldsymbol{u}(k-1) \end{bmatrix}; \boldsymbol{\gamma} \in \mathcal{R}^{2(N_2 - N_1 + 1), 1}$$
(21)

#### SIMULATION RESULTS

As a simulation example it was chosen a TITO nonminimum phase system given by equations (22)-(24). The continuous-time model of this system has only positive roots in the numerators of the transfer functions (22). The sampling period was chosen as 0.5 [s]. The step responses of the system are in Fig. 1 - 4.

$$\boldsymbol{G}(s) = \begin{bmatrix} \frac{-1.5s+1.5}{s^2+2s+1} & \frac{-s+1}{s^2+2s+2} \\ \frac{-0.5s+0.5}{s^2+2s+2} & \frac{-1.2s+1}{s^2+2s+1} \end{bmatrix}$$
(22)

$$\begin{array}{c} \alpha_{11}(z^{-1}) = 1 - 1.2722z^{-1} + 0.4151z^{-2}; \\ \alpha_{12}(z^{-1}) = 0.1879z^{-1} - 0.095z^{-2}; \\ \alpha_{21}(z^{-1}) = 0.0662z^{-1} - 0.0331z^{-2}; \\ \alpha_{22}(z^{-1}) = 1 - 1.2708z^{-1} + 0.4119z^{-2} \end{array}$$

$$(23)$$

$$\beta_{11}(z^{-1}) = -0.321z^{-1} + 0.551z^{-2};$$
  

$$\beta_{12}(z^{-1}) = -0.2019z^{-1} + 0.3455z^{-2};$$
  

$$\beta_{21}(z^{-1}) = -0.101z^{-1} + 0.1764z^{-2};$$
  

$$\beta_{22}(z^{-1}) = -0.2741z^{-1} + 0.4304z^{-2}$$
(24)



Figure 1: Step Functions of  $G_{11}(s)$  of TITO Model (22)



Figure 2: Step Functions of  $G_{12}(s)$  of TITO Model (22)



Figure 3: Step Functions of  $G_{21}(s)$  of TITO Model (22)



Figure 4: Step Functions of  $G_{22}(s)$  of TITO Model (22)

MPC of the non-minimum phase TITO system was implemented using MATLAB scripts. The optimization part was programmed using the Hildreth's dual method (Wang, 2009).

The previously introduced methods applied for control of SISO systems were not successful. In this case of the multivariable system undershoots were not eliminated, as can be seen in Fig. 5. The approach proposed in this paper provided satisfactory results, as can be seen in Fig. 6. The minimum, control and prediction horizons were chosen as  $N_1=1$ ,  $N_u=30$  and  $N_2=40$ . The constraints of variables were set according to (21) in several initial steps which cover the undershoot. In this particular case it was 10 steps.

The proposed method was applied for control of the introduced TITO systems with an elimination of significant undershoots in the initial part of the predictive control. The modification was active only during start-up of the control process. It is not appropriate for other step changes which also causes some minor undershoots.



Figure 5: Simulation of Control without Proposed Modification



Figure 6: Simulation of Control with Proposed Modification

#### CONCLUSIONS

The undershoot during the control of the non-minimum phase TITO system was eliminated using the proposed particular setting of constraints in the predictive control. This modification enables successful control of systems with this specific behaviour. The presented restriction is applied in the initial part of control. The approaches suitable for SISO systems were not satisfactory in case of the TITO system. The principal of the proposed modification is based on inequality constraints settings in the optimization task. The modification is used in several initial sampling periods in MPC and the undershoots are successfully eliminated.

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