TESTING HYPOTHESES USED IN ANALYSIS OF CONTROL QUALITY

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ABSTRACT

Simulation is an important tool for testing and verification of newly designed or modified control algorithms. One of the aims of the simulation verification is a comparison of control quality achieved with new or modified methods with control quality achieved with known methods. For an analysis of control quality, criterions based namely on sum of powers of control errors and sum of powers of control increments are commonly used. These criterions can result only in descriptive attributes of control quality. It means that on the basis of particular values of the criterions it is not possible to identify if the control quality achieved with one algorithm is statistically significantly different from control quality achieved with another algorithm. The aim of this paper is examining of control quality with use of testing hypotheses on existence of statistically significant differences between partial values of the control quality criterions in individual sampling periods. The analysis was performed on a strictly defined significance level 0.001, which is a standardly used value in technical applications. A realization is presented on a simulation of a multivariable predictive control with a modified optimization technique.

INTRODUCTION

In process control (Corriou, 2004), various approaches to a synthesis of controllers have been proposed with many particular applications e.g. (Abraham et al., 2018). Simulation is an important tool in technical applications e.g. (Samsonov et al., 2019; Strmiska et al., 2018), for a verification and testing of designed controllers e.g. (Kubalcik et al., 2006; Spacek, 2017). In simulations, among others, a quality of control achieved with individual controllers can be observed.

Achievement of a suitable quality of control is one of the main aims in the process control. The quality of control is then examined in order to evaluate which control algorithm reaches the best results in a particular control problem. The control algorithms which yield appropriate control results are often complex and computationally demanding. Therefore, there is an effort to simplify the control algorithms. This simplification is obviously at the expense of control quality. The quality of control is then examined in order to evaluate, if it was significantly affected or if it is still suitable for a particular control problem. In this paper, this case was considered.

As a suitable example of an improving a control algorithm with regards to decreasing of the computational complexity, a multivariable Model predictive control (MPC) will be considered. Model predictive control is one of currently utilized modern control methods, as can be seen in e.g. (Camacho et al., 2004; Rossiter, 2003).

In MPC, a computational complexity can significantly increase in control of multivariable processes, control of processes with fast dynamics and in case of higher horizons or constrained variables. A significantly important part of the constrained MPC is an optimization task. It is characterized by higher computational complexity. Therefore, a reduction of the computational complexity of the optimization methods in MPC has been widely researched. Various methods of improving of an optimization part of MPC can be seen in an explicit optimization solution with applications (Ingole et al., 2015). An alternative approach is a development of modifications of online optimization procedures, e.g. (Wang, 2009).

The control quality is often analyzed using general control quality criterions based on sums of powers of control increments and on sums of powers of control errors (Kubalcik et al., 2006). These criterions can result only in descriptive attributes of control quality. Therefore, on the basis of particular values of the criterions, it is not possible to identify if the control quality achieved with one algorithm is statistically significantly different from control quality achieved with another algorithm. The aim of this paper is to examine the control quality with use of testing hypotheses (Kitchenham et al., 2016; Vaclavik et al., 2019) on existence of statistically significant differences between partial values of the control quality criterions in individual sampling periods. The analysis was performed on a strictly defined significance level 0.001, which is a standardly used value in technical applications. In this paper, the testing hypothesis is applied in comparison of control quality between a standard MPC algorithm and particularly modified MPC algorithm (Kubalcik et al., 2019), where a modification of an iterative numerical optimization method was

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proposed. The modification consists in an addition of a new termination condition in the iterative algorithm.

MODEL OF CONTROLLED PROCESS

For purpose of discrete simulations, a model in the form of the matrix fraction (Kucera, 1991; Fajmon and Novak, 2010) with two-inputs and two outputs defined by (1)-(2) was considered.

$$\boldsymbol{G}(z^{-1}) = \begin{bmatrix} G_{11}(z^{-1}) & G_{12}(z^{-1}) \\ G_{21}(z^{-1}) & G_{22}(z^{-1}) \end{bmatrix} \tag{1}$$

$$\boldsymbol{G}(z^{-1}) = \boldsymbol{A}^{-1}(z^{-1})\boldsymbol{B}(z^{-1}) \tag{2}$$

The structure of the matrices $\boldsymbol{A}(z^{-1})$ and $\boldsymbol{B}(z^{-1})$ is described by (3)-(6).

$$\boldsymbol{A}(z^{-1}) = \begin{bmatrix} \alpha_{11}(z^{-1}) & \alpha_{12}(z^{-1}) \\ \alpha_{21}(z^{-1}) & \alpha_{22}(z^{-1}) \end{bmatrix} \tag{3}$$

$$\boldsymbol{B}(z^{-1}) = \begin{bmatrix} \beta_{11}(z^{-1}) & \beta_{12}(z^{-1}) \\ \beta_{21}(z^{-1}) & \beta_{22}(z^{-1}) \end{bmatrix} \tag{4}$$

$$\left.\begin{aligned} \alpha_{11}(z^{-1}) &= 1 + \alpha_{111}z^{-1} + \alpha_{112}z^{-2}; \\ \alpha_{12}(z^{-1}) &= \alpha_{121}z^{-1} + \alpha_{122}z^{-2}; \\ \alpha_{21}(z^{-1}) &= \alpha_{211}z^{-1} + \alpha_{212}z^{-2}; \\ \alpha_{22}(z^{-1}) &= 1 + \alpha_{221}z^{-1} + \alpha_{222}z^{-2} \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} \beta_{11}(z^{-1}) &= \beta_{111}z^{-1} + \beta_{112}z^{-2}; \\ \beta_{12}(z^{-1}) &= \beta_{121}z^{-1} + \beta_{122}z^{-2}; \\ \beta_{21}(z^{-1}) &= \beta_{211}z^{-1} + \beta_{212}z^{-2}; \\ \beta_{22}(z^{-1}) &= \beta_{221}z^{-1} + \beta_{222}z^{-2} \end{aligned}\right\} \tag{6}$$

MODEL PREDICTIVE CONTROL

The model predictive control (Camacho, 2004) has been widely implemented in control engineering practice. A predictive controller includes a model of the controlled process for purpose of a computation of predictions of the output variables. Constants N_1 and N_2 are minimum and maximum prediction horizons. In a receding horizon strategy (Rossiter, 2003), a horizon window is given by a maximum prediction horizon N_2. The manipulated variable is denoted as $\boldsymbol{u}(k)$, $\boldsymbol{y}(k)$ is the output controlled signal. The variable $\boldsymbol{e}(k)$ is a control error and $\boldsymbol{w}(k)$ is a reference signal.

A vector of future increments of the manipulated variable $\Delta\boldsymbol{u}$ with N_u elements is determined by solving an optimization task. N_u is a control horizon. The unknown variable $\boldsymbol{y}$ is then determined by prediction equations (9)-(11). Matrices $\boldsymbol{P}$ and $\mathcal{G}$ (10)-(12) contain a zero matrix $\boldsymbol{Z}$.

$$\left.\begin{aligned} \boldsymbol{A}(z^{-1})\boldsymbol{y}(k) &= \boldsymbol{B}(z^{-1})\boldsymbol{u}(k) + \Delta^{-1}(z^{-1})\boldsymbol{C}(z^{-1})\boldsymbol{e}_s(k) \\ \Delta(z^{-1}) &= \begin{bmatrix} 1-z^{-1} & 0 \\ 0 & 1-z^{-1} \end{bmatrix} \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} \boldsymbol{y}(k) &= \boldsymbol{A}_1\boldsymbol{y}(k-1) + \boldsymbol{A}_2\boldsymbol{y}(k-2) + \boldsymbol{A}_3\boldsymbol{y}(k-3) + \\ &+ \boldsymbol{B}_1\Delta\boldsymbol{u}(k-1) + \boldsymbol{B}_2\Delta\boldsymbol{u}(k-2); \\ \boldsymbol{A}_1 &= \begin{bmatrix} (1-\alpha_{111}) & -\alpha_{121} \\ -\alpha_{211} & (1-\alpha_{221}) \end{bmatrix}; \\ \boldsymbol{A}_2 &= \begin{bmatrix} (\alpha_{111}-\alpha_{112}) & (\alpha_{121}-\alpha_{122}) \\ (\alpha_{211}-a_{212}) & (\alpha_{221}-\alpha_{222}) \end{bmatrix}; \\ \boldsymbol{A}_3 &= \begin{bmatrix} \alpha_{112} & \alpha_{122} \\ \alpha_{212} & \alpha_{222} \end{bmatrix}; \boldsymbol{B}_1 = \begin{bmatrix} \beta_{111} & \beta_{121} \\ \beta_{211} & \beta_{221} \end{bmatrix}; \\ \boldsymbol{B}_2 &= \begin{bmatrix} \beta_{112} & \beta_{122} \\ \beta_{212} & \beta_{222} \end{bmatrix} \end{aligned}\right\} \tag{8}$$

$$\underbrace{\begin{bmatrix} \boldsymbol{y}(k+N_1) \\ \vdots \\ \boldsymbol{y}(k+N_2) \end{bmatrix}}_{\boldsymbol{y}} = \boldsymbol{P}\begin{bmatrix} \boldsymbol{y}(k) \\ \boldsymbol{y}(k-1) \\ \boldsymbol{y}(k-2) \\ \Delta\boldsymbol{u}(k-1) \end{bmatrix} + \mathcal{G}\underbrace{\begin{bmatrix} \Delta\boldsymbol{u}(k) \\ \Delta\boldsymbol{u}(k+1) \\ \vdots \\ \Delta\boldsymbol{u}(k+N_u-1) \end{bmatrix}}_{\Delta\boldsymbol{u}} \tag{9}$$

$$\left.\begin{aligned} \boldsymbol{P} &= \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} & \cdots & \boldsymbol{P}_{14} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} & \cdots & \boldsymbol{P}_{24} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{P}_{i1} & \boldsymbol{P}_{i2} & \cdots & \boldsymbol{P}_{i4} \end{bmatrix}; \\ \mathcal{G} &= \begin{bmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} & \cdots & \mathcal{G}_{1j} \\ \mathcal{G}_{21} & \mathcal{G}_{22} & \cdots & \mathcal{G}_{2j} \\ \vdots & & \ddots & \vdots \\ \mathcal{G}_{i1} & \mathcal{G}_{i2} & \cdots & \mathcal{G}_{ij} \end{bmatrix} \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} &\mathcal{G} \in \mathcal{R}^{2N_2\text{-}2N_1+2,2N_2}; \\ &\mathcal{G} = \boldsymbol{Z}; \boldsymbol{Z} \in \mathcal{R}^{2N_2\text{-}2N_1+2,2N_2}; \\ &\mathcal{G}_{11} = \mathcal{G}_{22} = \mathcal{G}_{33} = \boldsymbol{B}_1; \\ &\mathcal{G}_{21} = \mathcal{G}_{32} = (\boldsymbol{A}_1\boldsymbol{B}_1 + \boldsymbol{B}_2); \\ &\mathcal{G}_{31} = (\boldsymbol{A}_1^2\boldsymbol{B}_1 + \boldsymbol{A}_1\boldsymbol{B}_2 + \boldsymbol{A}_2\boldsymbol{B}_1); \\ &\left(\begin{aligned} \mathcal{G}_{i1} &= \boldsymbol{A}_1\mathcal{G}_{(i-1)1} + \boldsymbol{A}_2\mathcal{G}_{(i-2)1} + \boldsymbol{A}_3\mathcal{G}_{(i-3)1} \\ \mathcal{G}_{i(j-1)} &= \boldsymbol{A}_1\mathcal{G}_{(i-1)(j-1)} + \boldsymbol{A}_2\mathcal{G}_{(i-2)(j-1)} + \\ &+ \boldsymbol{A}_3\mathcal{G}_{(i-3)(j-1)} + \boldsymbol{B}_2 \\ \mathcal{G}_{ij} &= \boldsymbol{B}_1 \end{aligned}\right), \\ &i = 4,\dots,N_2; \; j = 1,\dots,i \end{aligned}\right\} \tag{11}$$

$$\left.\begin{array}{c} \boldsymbol{P} \in \mathcal{R}^{2N_2,8}; \\ \boldsymbol{P}_{11} = \boldsymbol{A}_1; \boldsymbol{P}_{12} = \boldsymbol{A}_2; \boldsymbol{P}_{13} = \boldsymbol{A}_3; \boldsymbol{P}_{14} = \boldsymbol{B}_2; \\ \boldsymbol{P}_{21} = \boldsymbol{A}_1^2 + \boldsymbol{A}_2; \boldsymbol{P}_{22} = \boldsymbol{A}_1\boldsymbol{A}_2 + \boldsymbol{A}_3; \\ \boldsymbol{P}_{23} = \boldsymbol{A}_1\boldsymbol{A}_3; \boldsymbol{P}_{24} = \boldsymbol{A}_1\boldsymbol{B}_2; \\ \boldsymbol{P}_{31} = \boldsymbol{A}_1^3 + \boldsymbol{A}_1\boldsymbol{A}_2 + \boldsymbol{A}_3 + \boldsymbol{A}_1\boldsymbol{A}_2; \\ \boldsymbol{P}_{32} = \boldsymbol{A}_1^2\boldsymbol{A}_2 + \boldsymbol{A}_1\boldsymbol{A}_3 + \boldsymbol{A}_2^2; \\ \boldsymbol{P}_{33} = \boldsymbol{A}_1^2\boldsymbol{A}_3 + \boldsymbol{A}_3\boldsymbol{A}_2; \boldsymbol{P}_{34} = \boldsymbol{A}_1^2\boldsymbol{B}_2 + \boldsymbol{A}_2\boldsymbol{B}_2; \\ \left(\boldsymbol{P}_{ij} = \boldsymbol{A}_1\boldsymbol{P}_{(i-1)j} + \boldsymbol{A}_2\boldsymbol{P}_{(i-2)j} + \boldsymbol{A}_3\boldsymbol{P}_{(i-3)j}\right), \\ i = 4,\dots,N_2; j = 1,\dots,i \end{array}\right\} \quad (12)$$

The optimization problem is then solved using quadratic programming. A cost function J is defined by (13)-(14) where the $\Delta\boldsymbol{u}$ vector is solved with regards to m constraints (15)-(16), where the matrix $\boldsymbol{I}$ is an identity matrix. Dimension n is considered as N_u.

$$J = (\boldsymbol{y} - \boldsymbol{w})^T(\boldsymbol{y} - \boldsymbol{w}) + \Delta\boldsymbol{u}^T\Delta\boldsymbol{u} \quad (13)$$

$$\left.\begin{array}{c} J = \frac{1}{2}\Delta\boldsymbol{u}^T\boldsymbol{H}\Delta\boldsymbol{u} + \boldsymbol{b}^T\Delta\boldsymbol{u}; \\ \boldsymbol{H} \in \mathcal{R}^{2N_u,2N_u}; \\ \boldsymbol{b} \in \mathcal{R}^{2N_u,1}; \\ \boldsymbol{H} = \mathcal{G}^T\mathcal{G} + \boldsymbol{I}; \\ \boldsymbol{b} = \mathcal{G}^T\left(\boldsymbol{P}\begin{bmatrix} \boldsymbol{y}(k) \\ \boldsymbol{y}(k-1) \\ \boldsymbol{y}(k-2) \\ \Delta\boldsymbol{u}(k-1) \end{bmatrix} - \boldsymbol{w}\right) \end{array}\right\} \quad (14)$$

$$\boldsymbol{M}\Delta\boldsymbol{u} \le \boldsymbol{\gamma} \quad (15)$$

$$\begin{array}{c} \boldsymbol{M} \in \mathcal{R}^{m,n},\ \boldsymbol{\gamma} \in \mathcal{R}^{m,1} \\ \boldsymbol{M} = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{m1} & M_{m2} & \cdots & M_{mn} \end{bmatrix} \\ \boldsymbol{\gamma} = [\gamma_1 \ \ \gamma_2 \ \ \cdots \ \ \gamma_m]^T \end{array} \quad (16)$$

MODIFIED ALGORITHM OF MODEL PREDICTIVE CONTROL

The Hildreth method (Wang, 2009) is a widely applied numerical method for the quadratic programming optimization. In this optimization method, numerical iterations are considered as particular sub-results $\boldsymbol{d}$, which are gradually improving into the final solution (17).

$$\Delta\boldsymbol{u}(k) = -\boldsymbol{H}^{-1}(\boldsymbol{M}^T\boldsymbol{d} + \boldsymbol{b}^T) \quad (17)$$

Vector $\boldsymbol{d}$, which is being improved during the Hildreth's method, can be expressed by (18). This vector is being progressively improved in each ω-th iteration.

$$\boldsymbol{d}(\omega) = \left[d_1(\omega) \ \ \cdots \ \ d_{N_u}(\omega)\right]^T \quad (18)$$

The numerical algorithm is terminated if there are lower differences between sub-results from two previous iterations than a defined threshold.

The modification published in (Kubalcik et al., 2019) incorporated a new termination condition based on inequality (19). The computation is terminated when the solution fulfils constraints of variables.

$$\boldsymbol{M}\left[-\boldsymbol{H}^{-1}(\boldsymbol{M}^T\boldsymbol{d}(\omega) + \boldsymbol{b}^T)\right] \le \boldsymbol{\gamma} \quad (19)$$

As it was proved by simulations, application of this condition significantly saved computational time. On the other hand, the solution of the optimization task might not be optimal and therefore the quality of control might be significantly decreased. Further in this paper, testing hypothesis was applied to compare control quality of the modified and non-modified algorithms. The result of the testing should be a decision if the difference in control quality according to a chosen criterion is statistically significant or not.

PROPOSAL OF ANALYSIS OF CONTROL QUALITY USING STATISTICAL METHODS

In this paper, testing hypotheses (Kitchenham et al., 2016) is used for a more detailed comparison of the control quality of the modified and non-modified optimization algorithms in MPC. Testing hypotheses based on an analysis of existence of statistically significant differences between variables appears to be more suitable for purpose of evaluation of control quality achieved with different control algorithms than evaluation of control quality from single values of control quality criterions (20)-(21). The result of the testing should be a decision if the difference in control quality according to a chosen criterion is statistically significant or not.

An assessment of control quality of modified algorithm can be based on comparison of pairs of data of control quality criterions (20)-(21) before and after applied modification in each sampling period of control. For each criterion, parts (22)-(23) of the criterions are paired-compared during the whole control process. Testing hypotheses on an existence of statistically significant differences can provide a conclusion with regards to a chosen significance level. In this paper, the strictly defined significance level 0.001, which is a standardly used value in technical applications, was used.

$$J_1 = \sum_k \left[\Delta u_1(k)\right]^2 + \sum_k \left[\Delta u_2(k)\right]^2 \quad (20)$$

$$J_2 = \sum_k [w_1(k) - y_1(k)]^2 + \sum_k [w_2(k) - y_2(k)]^2 \quad (21)$$

$$J_1(k) = [\Delta u_1(k)]^2 + [\Delta u_2(k)]^2 \quad (22)$$

$$J_2(k) = [w_1(k) - y_1(k)]^2 + [w_2(k) - y_2(k)]^2 \quad (23)$$

In selection of a suitable method for testing of the data, testing normality (Vaclavik et al., 2019) of the data is important. A volume of available data is also an important aspect for selection of an appropriate method. For cases with less extensive data sets, exact tests are recommended (Kitchenham et al., 2016; Barot and Krpec, 2019). Each hypothesis is generally defined by the zero and alternative hypotheses.

Modern statistical approaches and software solutions, e.g. PAST Statistics (Hammer et al., 2000), are able to perform testing hypothesis in form of p value. If the p value is greater or equal to the defined significance value 0.001, the zero hypothesis is failed to rejected. In the opposite case, the zero hypothesis is rejected in favour of an alternative hypothesis. According to the testing normality, e.g. using Shapiro-Wilk test (Vaclavik et al., 2019) of data set, appropriate corresponding methods should be used for testing paired differences. In case of appearance of data normality, Paired T-test should be applied. In the opposite case, Wilcoxon Paired test should be used with respect to an exact variant for a particular volume of measured data.

SIMULATION RESULTS

For the purpose of comparison, MPC of TITO process (2) with polynomial matrices (24)-(25) both with and without the proposed modification was simulated in MATLAB. Constraints of the manipulated variables and increments of the manipulated variables were considered which is obvious from definition (27). Setting of constraints is obvious from (28) where $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{E}$ is a unit matrix.

$$\boldsymbol{A}(z^{-1}) = \begin{bmatrix} 1\text{-}1.3264\, z^{-1} + 0.3271\, z^{-2} & 0.024\, z^{-1} - 0.0029\, z^{-2} \\ \text{-}\,0.0711\, z^{-1} + 0.0759\, z^{-2} & 1\text{-}1.0911\, z^{-1} + 0.134\, z^{-2} \end{bmatrix} \quad (24)$$

$$\boldsymbol{B}(z^{-1}) = \begin{bmatrix} 0.2983\, z^{-1}\text{-}\,0.097\, z^{-2} & 0.093\, z^{-1} + 0.0682\, z^{-2} \\ 0.1755\, z^{-1} + 0.0688\, z^{-2} & 0.1779\, z^{-1} + 0.1065\, z^{-2} \end{bmatrix} \quad (25)$$

$$u_{\min} = -1.7,\ u_{\max} = 1.75,\ \Delta u_{\max} = 0.07 \quad (26)$$

$$\boldsymbol{M} = \begin{bmatrix} -\boldsymbol{E}^{2,2} & 0 & \cdots & 0 \\ -\boldsymbol{E}^{2,2} & -\boldsymbol{E}^{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{E}^{2,2} & -\boldsymbol{E}^{2,2} & \cdots & -\boldsymbol{E}^{2,2} \\ \hline \boldsymbol{E}^{2,2} & 0 & \cdots & 0 \\ \boldsymbol{E}^{2,2} & \boldsymbol{E}^{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{E}^{2,2} & \boldsymbol{E}^{2,2} & \cdots & \boldsymbol{E}^{2,2} \\ \hline \boldsymbol{I}^{2,2} & 0 & \cdots & 0 \\ 0 & \boldsymbol{I}^{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{I}^{2,2} \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} \boldsymbol{u}(k-1) + \begin{bmatrix} u_{\min} \\ u_{\min} \end{bmatrix} \\ \vdots \\ \boldsymbol{u}(k-1) + \begin{bmatrix} u_{\min} \\ u_{\min} \end{bmatrix} \\ \hline -\boldsymbol{u}(k-1) + \begin{bmatrix} u_{\max} \\ u_{\max} \end{bmatrix} \\ \vdots \\ -\boldsymbol{u}(k-1) + \begin{bmatrix} u_{\min} \\ u_{\min} \end{bmatrix} \\ \hline \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix} \quad (27)$$

For the purpose of a control quality analysis, control quality criterions (20)-(21) and their parts (22)-(23) were evaluated. The results for various values of N (maximum prediction and control horizon) can be seen in Table 1.

Table 1: Results of Control Quality Criterions for MPC without and with Modification

	MPC without Modif.	MPC with Modif.	MPC without Modif.	MPC with Modif.
N	J_1	J_1	J_2	J_2
10	58,3219	58,3219	195,4043	195,4043
15	56,0261	56,0260	196,0840	196,0841
20	55,0907	55,0907	197,6620	197,6621
25	55,6287	55,6286	196,7332	196,7333
30	58,1919	58,1917	193,0959	193,0962
35	58,2704	58,2704	192,9479	192,9479
40	58,3068	58,3068	192,7495	192,7495

As it is obvious from the results in Table 1, the values of the criterions for the modified and original MPC differs by one percent at the most. Better control quality was obviously achieved with the original MPC algorithm.

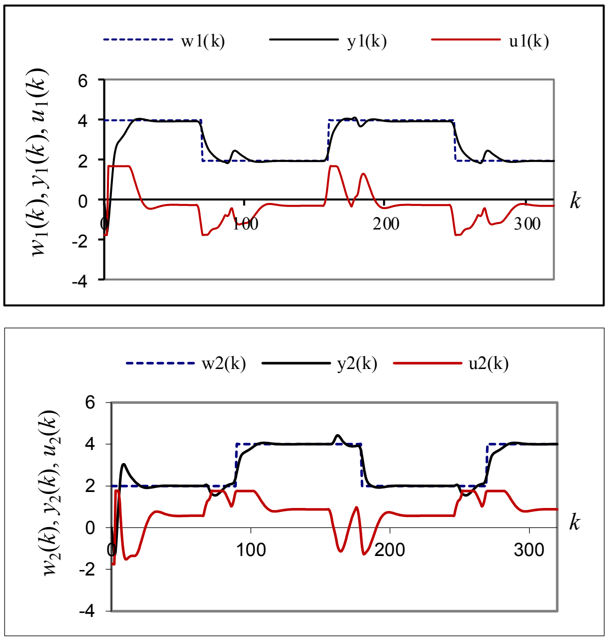

Figure 1: Simulation of MPC without Modifications

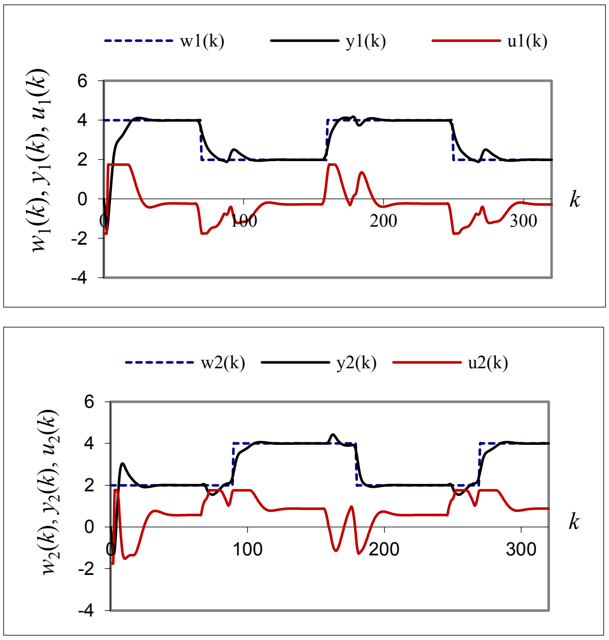

Figure 2: Simulation of MPC with Modification

A complexity function for the non-modified algorithm is expressed by equation (28). Equation (29) expresses the complexity function for the modified algorithm. Equations (28) and (29) were computed using a non-linear regression (Krivy et al., 2000). Only the highest power of the variable N is displayed in the equations.

$$\left.\begin{array}{l} O(N) = 21859N^3 \approx 21859N^3 - \\ -14373N^2 + 6575.9N + 681 \end{array}\right\} \quad (28)$$

$$\left.\begin{array}{l} O(N) = 15328N^3 \approx 15328N^3 + \\ +474011N^2 - 1\times10^7 N + 6\times10^7 \end{array}\right\} \quad (29)$$

More relevant conclusions concerning the control quality can bring testing hypothesis. The results of testing hypothesis, which means a corresponding conclusion (failed to reject or rejected), with strictly defined significance level 0.001 are in Table 2.

Table 2: Results of Testing Hypotheses on Non-Existence of Statistical Significant Differences Using Paired Wilcoxon Exact Test on Sig. Level 0.001

	MPC without Modif.	MPC with Modif.	MPC without Modif.	MPC with Modif.
N	Data of J_1	Data of J_1	Data of J_2	Data of J_2
10	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
15	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
20	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
25	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
30	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
35	Fail to Reject on α=0.001		Fail to Reject on α=0.001	
40	Fail to Reject on α=0.001		Fail to Reject on α=0.001	

For testing hypotheses, the Wilcoxon exact test was used with respect to failing normality of data, which was determined using the Shapiro-Wilk test. Both the testing normality and testing hypotheses was performed using the PAST Statistics software (Hammer, 2001). It was achieved the proof of non-existence of statistically significant differences in control quality according to the chosen criterions for each value of the problem dimension N. It was then proved that the application of the modified control algorithm, which significantly decreases the computational complexity, does not significantly influence control quality in comparison with the control quality achieved using the non-modified algorithm.

CONCLUSIONS

In the simulation of multivariable predictive control, the control quality achieved with two different algorithms was analyzed and compared using testing hypothesis. In this hypothesis testing, partial values of control quality criterions were analyzed in each sampling period of MPC. It would not be possible to consider statistical significance of differences in achieved control quality only from the descriptive attributes given by standardly used control quality criterions. The analysis using testing hypothesis was performed on a strictly defined significance level 0.001, which is a standardly used value in technical applications. Therefore, the achieved results had relevant informational value based on mathematical statistics. The control results of the original method were compared with the results obtained using its modification. The modification is in fact a simplification of the original method and it is supposed that the quality of control will be decreased. Nevertheless, it was then proved that the difference between control quality of modified and original methods of optimization in predictive control was not statistically significant.

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