MATHEMATICAL MODEL OF BIOMIMETIC UNDERWATER VEHICLE

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ABSTRACT

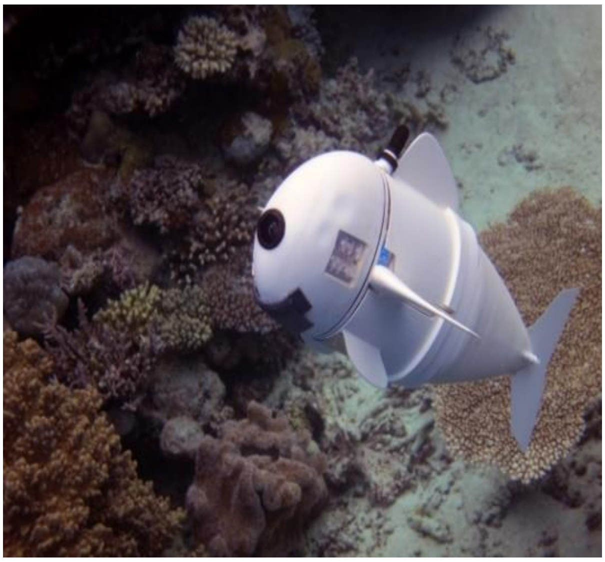
High-rate growth of underwater robotics has led to the development of new robot models that move underwater mimicking marine organisms' motion. They are called "biomimetic underwater vehicles" (BUV) because they use the wave propulsion modelled after fish shapes and kinematics.

The paper analyses a mathematical model of the BUV motion. The model was implemented in Matlab program for numerical tests and simulation of a vehicle's performance under real conditions. This way the most effective way to change the BUV course was determined.

INTRODUCTION

In recent years, dynamic development of biomimetic unmanned vehicles (BUV) has been observed. In many countries research and scientific institutions, such as Massachusetts Institute of Technology (MIT) and the Defense Advanced Research Projects Agency (DARPA) in the USA, work on designs and upgrades of BUVs that have unique motoric capabilities in the marine, land and air environments due to their mimicking of natural movement of animals, including fish, birds and insects. Fig. 1 (Katzschmann, DelPreto, MacCurdy and Rus 2018).

One of the latest design trends in this field is the development of underwater robots, in particular BUVs (Vincent 2003, Szymak 2016). The vehicle designs are usually modelled on fish, which are the prototype of the shape and movement, although vehicles modelled on jellyfish, rays, marine mammals, etc. were recently noted. Regardless of the prototype of the vehicle engineering, in each case a thorough analysis of the movement of the animal is necessary as well as its simplified mathematical modelling. In contrast to screw-propelled ROVs (Remotely Operated Vehicles) and AUVs Autonomous Underwater Vehicles), BUVs are wave-driven by a fins-like motion.



Figures 1: MIT's biomimetic underwater vehicle (BUV) "SoFi"

The next chapter presents a mathematical model developed for the BUV motion modelling. Then, selected results are presented of the BUV maneuverability simulation tests on a mini CyberSeal vehicle (Fig. 2) performed at the Institute of Electrical Engineering and Automatics of the Polish Naval Academy the AMW. The last chapter summarizes the study.

Figures 2: Mini CyberSeal

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MATHEMATICAL MODEL

Mathematical modelling of an underwater vehicle is a complex issue (Szymak 2016). This is due to the difficulty of experimental determination or calculation of a very large number of parameters that must be known to solve motion equations. Therefore, in order to simulate the motion of an underwater vehicle, some simplifications regarding the vehicle design are accepted, e.g.: it is considered as a rigid body, with 3 symmetry planes, its centre of mass coincides is in its centre of buoyancy, it moves with six degrees of freedom, at a low speed, in a viscous fluid.

By analysing an underwater vehicle's motion, two reference systems are defined:

1) Earth-fixed coordinate system xyz

2) mobile, underwater vehicle-fixed coordinate system $x_o y_o z_o$.

The mobile coordinate system is called the "vehicle reference system" and it begins in the vehicle's geometric centre. The axes of this coordinate system correspond to:

1) x_0 – longitudinal axis from stern to bow

2) y_0 – transverse axis to starboard side

3) z_0 – vertical axis to bottom.

Movement of the mobile coordinate system xoyozo is reported relative to the Earth-fixed coordinate system xyz. Because of the vehicle's low speed, the acceleration of points on the Earth's surface due to its spinning motion is neglected and the xyz system is considered as stationary. It is suggested that the angular and linear velocities be described in the vehicle reference system, while the vehicle's orientation should be described in the stationary coordinate system. The vehicle motion parameters are defined according to the SNAME notation, as shown in Table 1.

Table 1: Notation used to describe underwater vehicle motion

Degrees of freedom	**Motion**	**Forces and moments**	**Angular and linear velocities**	**Position and Euler angles**
1	Along x_o axis (from stern to bow)	X	u	x
2	Along y_o axis (to starboard side)	Y	v	y
3	Along z_o axis (from up to down)	Z	w	z
4	Around x_o axis (rolling)	K	p	ϕ
5	Around y_o axis (rocking)	M	q	θ
6	Around z_o axis (yawing)	N	r	ψ

Considering the above assumptions, a non-linear model of motion with six degrees of freedom was adopted for the mini CyberSeal's motion simulation. The vehicle motion was described by six differential equations with the following matrix form:

$$\dot{v} \quad \boldsymbol{D}(v)v \quad \boldsymbol{g}(\eta) \qquad (1)$$

where:

v – vector of linear and angular velocities, i.e. v = [u, v, w, p, q, r]

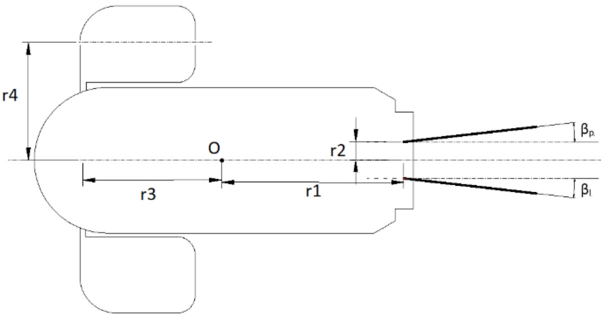
– vector of vehicle position coordinates and Euler's angle in the fixed system

M – inertia matrix (sum of rigid body matrix M_{RB} and associated masses matrix M_A)

D(ν) – hydrodynamic damping matrix

g(η) – restoring forces matrix (gravity P and buoyancy B)

– vector of forces and moments affecting the vehicle



Figures 3: Mini CyberSeal propulsion model

The left side of equation (1) describes forces and moments of forces induced by physical phenomena, such as: rigid body inertia and inertia of masses associated with viscous liquid, hydrodynamic resistance of water, balance of gravity and buoyancy.

The right side of equation (1) shows the vector of forces and moments of forces acting on the vehicle, generated by the vehicle's propulsion system. In addition, the right side of equation (1) considers the impact of environmental disturbances (wind, wave and sea current, the largest impact on BUV). Using the mathematical relationships reported in references (Fossen 1994), parameters can be calculated of the matrix describing the left side of equation (1). To analyse the right side of equation (1), however, more compiled calculations should be adopted. Commonly used underwater vehicles are mostly screw-propelled, whereby with specific mathematical formulas (Katzschmann, DelPreto, MacCurdy and Rus 2018, Fossen 1994) the pressure generated by the propulsor can be calculated. With the new propulsion system, i.e. the wave-propulsion mimicking the action of fish fins, the forces and moments of forces should be determined τ_P:

(2)

where:

X, Y, Z – forces acting on vehicle in the longitudinal, transverse and vertical symmetry axis, respectively

K, M, N – moments of forces acting in relation to the longitudinal, transverse, and vertical symmetry axis, respectively.

The vector of forces and moments of forces generated by the wave propulsion can be calculated by considering the propulsion system set-up in each design. Fig. 3 shows the mini CyberFoka propulsion model consisting of two counterphased tail fins and two independently controlled side fins.

The thrust produced by each fin should be brought to the centre of gravity O (Fig. 3) using simple vector transformation formulas:

$$X = X_{tl} + X_{tp} + X_l + X_p \tag{3}$$

$$Y = Y_{tl} + T_{tp} \tag{4}$$

$$Z = Z_l + Z_p \tag{5}$$

$$K = 0 \tag{6}$$

$$M = M_l + M_p \tag{7}$$

$$N = N_{tl} - N_{tp} + N_l - N_p \tag{8}$$

where:

tl, tp, l i p – subscripts referring to the action of the left rear fin, right rear fin, left side fin and right-side fin, respectively.

The individual components of the vector, e.g. . X_{tl}, Y_{tl}, N_{tl} can be calculated using the position of these fins with respect to the centre of gravity according to equation:

(9)

(10)

(11)

(12)

(13)

(14)

(15)

(16)

(17)

(18)

where:

T_{tl}, T_{tp}, T_l, T_p – thrusts generated by the left rear fin, right rear fin, left side fin and right-side fin, respectively

β_l, β_p, α_l, α_p – angles shown in Fig. 3,

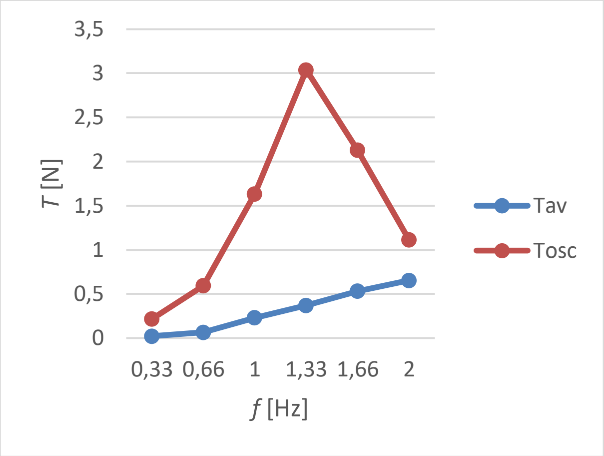
r_1 – distance from vehicle's centre of gravity to the centre of rear fins' axes along x_0 axis,

r_2 – distance from rear fin axes centre to the centre of rear fins' rotation along y_0 axis,

r_3 – distance from vehicle's centre of gravity to the centre of side fins' rotation along x_0 axis,

r_4 – distance from side fins' rotation centre to the centre of left or right-side fin's rotation along y_0 axis.

The vector components for the right fin X_p, Z_p, M_p, N_p can be calculated from formulas (15-18) for the left side fin by inserting the right fin's inclination angle α_p.



Figures 4: Changes in average thrust T_{av} and in trust oscillation amplitude T_{osc} for different fin drive frequencies.

Thrusts T_{tl}, Tt_p, T_l, T_p generated by both rear fins and both side fins, respectively, were determined experimentally for precisely defined fin shapes and surfaces, using the laboratory bench described in the paper by P. Szymak and S. Bruski Microprocessor system for measurement of a thrust generated by an underwater vehicle (Szymak and Bruski 2016). Each fin's thrust is an instantaneous value changing in time, depending on the fin motion control parameters, i.e. fin oscillation amplitude and frequency. The thrusts depend also on the fin material type (stiffness). Therefore, at each change of the fin shape, dimensions and material the measurements should be repeated (Szymak and Bruski 2016) and the thrust experimentally evaluated. Based on the results of the measurements shown in Fig. 4, it can be assumed that thrust T generated by the fin is the sum of two components:

(19)

where:

T_{av} – constant thrust component at a specific fin oscillation frequency

T_{osc} – variable component modelled by a sinusoidal wave with a specific amplitude (at a specific fin oscillation frequency).

The mathematical model of the mini CyberFoka BUV wave propulsion was implemented in the Matlab environment. Such a model can be used to check many parameters, such as the best way to manoeuvre the BUV.

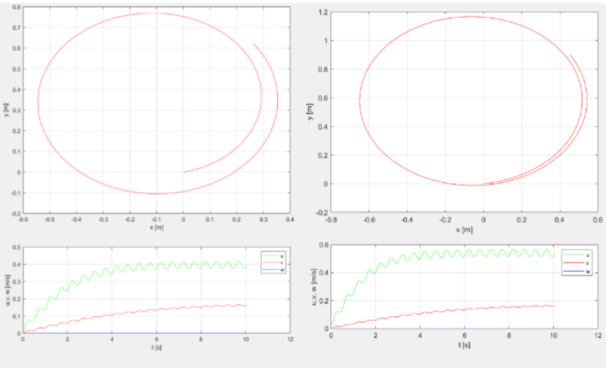
SIMULATION TEST RESULTS

The mathematical model presented in the previous chapter was used to check the BUV manoeuvrability. The dimensions of an actual BUV model were used for the tests. It was assumed that the underwater vehicle is made of a cylinder-shaped symmetrical hull, 0.44 m long and with 0.12 m diameter. The fin drives were positioned in relation to the centre of gravity as follows:

1) r1 = 0.11 m
2) r2 = 0.015 m
3) r3 = 0.08 m
4) r4 = 0.085 m

The following vehicle course changes were tested:

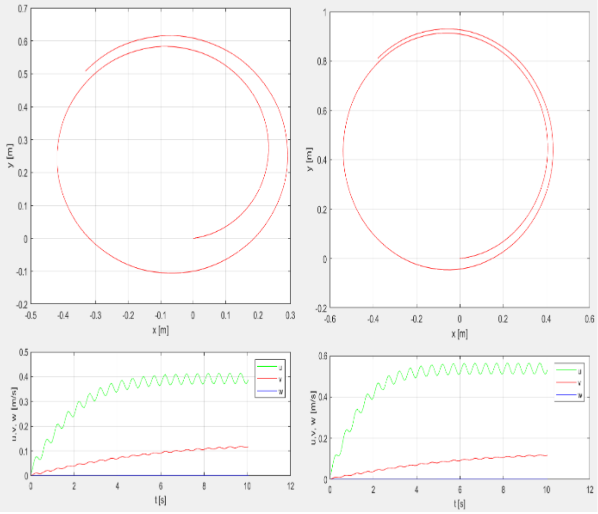
1) Both rear fins oscillate, one inclined by 45 degrees and the other in the neutral position, side fins do not work.
2) Both rear fins oscillate, one inclined by 45 degrees and the other by 15 degree in the same direction, side fins do not work.
3) Both rear fins oscillate, one inclined by 45 degrees and the other in the neutral position, side fins work in the neutral position.
4) Both rear fins oscillate, one inclined by 45 degrees and the other by 15 degree in the same direction, side fins work in the neutral position.



Figures 5: Changes in position on xy plane and in u and v velocities of a BUV with fins oscillating at 2 Hz in example 1 on the left side, in example 3 on the right side.

Fig. 5 shows changes in the position and in *u* and *v* velocities of BUV with fins oscillating at 2 Hz at the course changes No. 1 and 3. In the first case, thereby only the rear fins worked, with one inclined at 45 degrees and the other one in neutral position, the circulation radius was obtained of ca. 0.45 m. With the same rear fins' settings and with the side fins' additional operation, the BUV moved faster but along a trajectory with a larger circulation radius.

The next Fig. 6 shows results of the simulation of change in BUV position's on xy plane and in u and v velocities at the course changes No. 2 and 4, where the second rear fin was also inclined by 15 degrees in the same direction as the first fin. In both cases, smaller circulation radii were obtained, whereas course change No. 2 involving the rear fins' inclination, by 45 and 15 degrees, respectively, in the same direction, with non-working side fins, which turns out to be the most effective, i.e. with turning radius of 0.35 m.



Figures 6: Changes in position on xy plane and in u and v velocities of a BUV with fins oscillating at 2 Hz in example 2 on the left side, in example 4 on the right side.

SUMMARY

The mathematical model of a biomimetic underwater vehicle presented and implemented in the Matlab environment allows to perform initial simulations of the vehicle motion and to check the effectiveness of course changes depending on the inclinations of individual rear fins and the operation of the propulsion system with or without additional side fins. In the next stage, the results of simulation tests will be verified on a real object and other herein non-described course change combinations will be. After positive verification of the mathematical model or after possible modifications, further work is planned on the mathematical model for the development and subsequent testing of various BUV course and trim controllers.

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