APPROXIMATE ANALYSIS OF THE LIMITED RESOURCES QUEUING SYSTEM WITH SIGNALS

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ABSTRACT

Queuing systems with limited resources, in which customers require a device and a certain amount of limited resources for the duration of their service, proved their effectiveness in the performance analysis of modern wireless networks. However using queuing systems leads to complex calculations. In this paper, we propose a method for approximate analysis of queuing systems with limited resources and signals that trigger resource reallocation of customers. The results of approximate analysis were compared with simulations of the initial model.

I. INTRODUCTION

Queuing systems with limited resources with signals can be used to analyze the performance of contemporary wireless networks [1, 2]. Signal arrival indicates that a different amount of resources is required for the request. Upon arrival of a signal, a customer leaves the system and immediately comes again with a new resource requirements.
Analytical calculations of probabilistic indicators of resource queuing system with signals in case of Poisson arrivals are presented in [3, 4, 5]. Application of such analytical formulas to calculate the stationary characteristics is rather complicated, since it implies calculation of multiple convolutions of resources requirements cumulative distribution function (CDF), and with an increase in the dimension of the system, the duration of calculations increases too.
Analytical calculations for such systems require significant computing power, therefore, in [6, 7] we have developed the simulation tool for queuing systems with limited resources with signals.
Calculations with the help of simulation tools managed to reduce the load on computing resources in the calculations of the probability-time characteristics.
In the paper, we propose to consider an approximate method for calculating stationary characteristics of the model. Besides, we provide comparison of calculations accuracy with the methods, proposed in [3, 4].
The rest of the paper is organized as follows. Section II gives a brief description of the queuing system with limited resources with signal arriving, while section III describes approximation method for the considered model. In Section IV, the numerical evaluation of approximate method is performed, the results are compared with simulations. Section V concludes the paper.

II. QUEUING SYSTEM WITH SIGNALS

We consider a multiserver queueing system with N servers, in which arriving customer occupies not only a server, but also a volume of limited resources. The total volume of resources in the system is R and volumes of customers' resource requirements are independent identically distributed random variables with CDF $F_1(x)$. Here and further we assume discrete resources. In cases of continuous resource requirements the simplification algorithm which described in [5] can be applied. Customers arrive according to the Poisson process with intensity λ and the service times have exponential distribution with rate μ. Each customer in the system produces a flow of signals. Signals arrive according to the Poisson distribution with intensity γ. When a signal arrives, the customer releases the server and occupied resources and goes to the system again with new resource requirements with CDF $F_2(x)$. It follows that customers leave the system with intensity $\mu+\gamma$ and returns back with probability $\frac{\gamma}{\mu+\gamma}$.
Let $\xi(t)$ be the number of customers in the system at moment $t>0$ and $\gamma(t)=\left(\gamma_1(t),....,\gamma_{\xi(t)}(t)\right)$ - the vector of occupied resources by each customer. If $\xi(t)=0$, vector $\gamma(t)$ is empty. The system behavior is described by the stochastic process $X(t)=\left(\xi(t),\gamma(t)\right)$ over the set of states

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$$S=\left\{(n,r_1,...,r_n):0\le n\le N, r_i\ge 0, \sum_{i=1}^{n} r_i \le R\right\}.$$

Figure 1 shows the scheme of the queuing system.

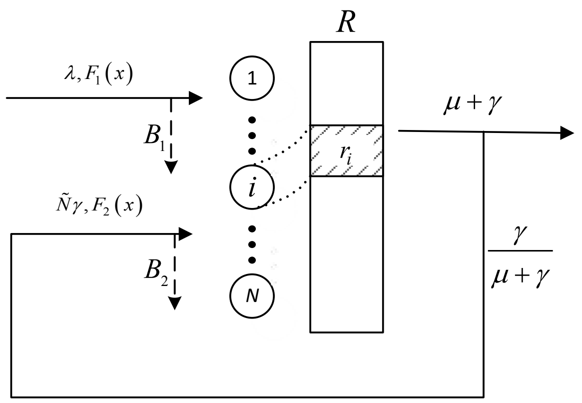

Figure 1. Resource queueing system scheme

In [4], system of equilibrium equations and stationary probabilities for this system were derived by means of matrix-geometric methods. With increasing of state space, dimension of generator matrix increases too and the calculations in real system scale becomes too complicated. In the next section, we tried to solve this problem with approximation method.

III. ASSUMPTION FOR APPROXIMATE ANALYSIS

We consider that customers which arrive with signal contain a new type of customers with intensity $\tilde{N}\gamma$, $\tilde{N}$ - average number of customers in the system. Denote $L=2$ as set of customers' type.

We assume here that resource requirements are independent of arrival and serving processes.

Denote $p_{l_k,r_{l_k}}$ the probability that k-th customer of l-type will require r_{l_k} resources and p_{l,r_l} the probability that a customer of l-type will require r_l resources. Then $p^{(k_l)}_{l,r_l}$ is the probability that k_l customers of l-type will require r_l, where $p^{(k_l)}_{l,r_l}$ is k-fold convolution of probabilities p_{l,r_l}. Denote $\rho_1=\dfrac{\lambda}{\mu+\gamma}$ - the offer load for customers of 1-st type and $\rho_2=\dfrac{\tilde{N}\gamma}{\mu+\gamma}$ - the offer load for customer of 2-nd type. According to [5], we can unite offer loads $p_r=\sum_{l=1}^{L}\dfrac{\rho_l}{\rho}p_{l,r_l}$, where $\rho=\sum_{l=1}^{L}\rho_l$ and calculate stationary probablites as follows:

$$q_{k,r}=q_0\frac{\rho^k}{k!}p_r^{(k)}, \quad (1)$$

where $p_r^{(k)}$ is k - fold convolution of propabilities p_r.

$$q_0=\left(\sum_{k=0}^{N}\sum_{j=0}^{R}\frac{\rho^k}{k!}p_j^{(k)}\right)^{-1} \quad (2)$$

Formula (2) defines normalization constant $G(n,r)$. In [5], we developed the recurrent algorithm for evaluation of normalization constant, which can be calculated by formula (3):

$$G(n,r)=G(n-1,r)+ \\ +\frac{\rho}{n}\sum_{i=0}^{r}p_i\left(G(n-1,r-i)-G(n-2,r-i)\right), \\ 2\le n\le N \quad (3)$$

with initial values:

$$G(1,r)=1+\rho\sum_{j=0}^{r}p_j, 0\le r\le R \quad (4)$$

Also in [5], given the normalization constant $G(n,r)$, we derived formulas for the blocking probability B, the average number of occupied resources b, the blocking probability of customers with type l - B_l.

$$B=1-G^{-1}(N,R)\sum_{j=0}^{R}p_jG(N-1,R-j); \quad (5)$$

$$B_l=1-G^{-1}(N,R)\sum_{n=0}^{N-1}\frac{\rho^n}{n!}\sum_{j=0}^{R}\sum_{i=0}^{j}p_{l,i}p_{j-i}^{(n)}= \\ =1-G^{-1}(N,R)\sum_{j=0}^{R}p_{l,j}G(N-1,R-j); \quad (6)$$

$$b=R-G(N,R)^{-1}\sum_{j=1}^{R}G(N,R-j); \quad (7)$$

In our model offered load for customers of 2-nd type depends on $\tilde{N}$, so we can calculate it by the formula bellow, again using the normalization constant:

$$\tilde{N}=\sum_{n=1}^{N}\sum_{r=0}^{R}nq_{n,r},$$

$$\tilde{N}=q_0\sum_{n=1}^{N}n\frac{\rho^n}{n!}\sum_{r=0}^{R}p_r^{(n)}=q_0\rho\sum_{n=1}^{N}\frac{\rho^{n-1}}{(n-1)!}\sum_{r=0}^{R}p_r^{(n)}= \\ =q_0\rho\sum_{n=0}^{N-1}\frac{\rho^n}{n!}\sum_{r=0}^{R}p_r^{(n+1)}=q_0\rho\sum_{n=0}^{N-1}\frac{\rho^n}{n!}\sum_{r=0}^{R}\sum_{j=0}^{r}p_jp_{r-j}^{(n)}= \\ =q_0\rho\sum_{n=0}^{N-1}\frac{\rho^n}{n!}\sum_{j=0}^{R}p_j\sum_{r=j}^{R}p_{r-j}^{(n)}=q_0\rho\sum_{j=0}^{R}p_j\sum_{n=0}^{N-1}\frac{\rho^n}{n!}\sum_{r=j}^{R-j}p_r^{(n)}= \\ =q_0\rho\sum_{j=0}^{R}p_jG(N-1,R-j). \quad (8)$$

Figure 2 shows the approximation algorithm for calculating performance metrics. Before start we must define min_dif – the accuracy threshold for average number of customers $\tilde{N}$. At the beginning of approximation, we have only first type of customers. We calculate $\tilde{N}$ with first type of customer and denote it as $prev\_\tilde{N}$. Having $\tilde{N}$, one can calculate input parameters for second type of customers, and calculate $\tilde{N}$ for two flows of customers with intenstites λ and $prev\_\tilde{N}\gamma$. On the next step we compare the difference between $\tilde{N}$ and $prev\_\tilde{N}$ according to the accuracy threshold min_dif. If the difference less then min_dif we calculate blocking probability of newly arriving customers B_1, blocking probability of second-type customers B_2 and average amount of occupied resources b, else return to denoting $\tilde{N}$, and calculate it with new intensity of the second flow.

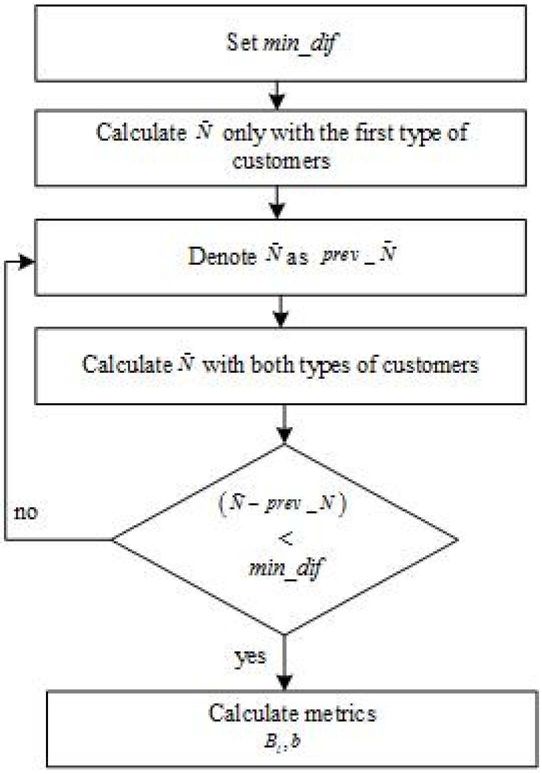

Figure 2. Block-scheme for approximation calculation

Note that blocking probability of the second-type customers can be interpret as the probability that signal arrival leads to the drop of a customer. In most practical cases, instead of it we need the probability B_3 that a customer is terminated before it finishes its service properly:

$$B_3 = \frac{\gamma \tilde{N} B_2}{\lambda \left(1 - B_1\right)}. \tag{9}$$

IV. APPROXIMATION RESULTS

In this section we compare approximation method results and the simulation results. We set the number of devices $N = 100$, the number of resources $R = 200$ and intensities $\lambda = 50$, $\mu = 1$ $\gamma = 5$. We used geometric distribution for resource requirements with parameter P_i for each type of customers'. $P_1 = 0.75$ for the newly arriving customers and $P_2 = [0.4; 0.8]$ for secondary customers.

The simulation results were compared with calculations by formulas (5-7), where performance measures of the queueing system were obtained using the geometric CDF and applying formulas (5-7) for discrete resource requirements (Figure 3, Figure 4, Figure 5).

One can note that the difference between analytic and simulation results increase with increase of average resource requirements after signal arrival. The main problem is in the method of generating signals. If in the initial model, upon arrival of a signal the customer releases occupied

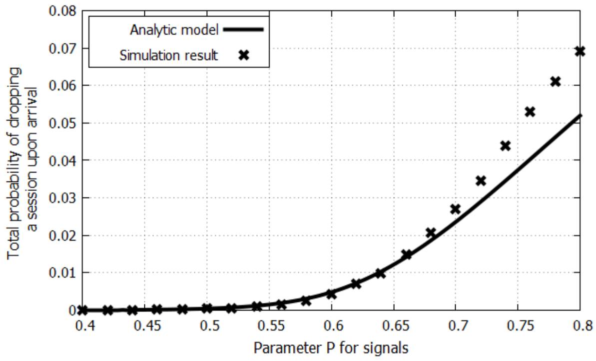

Figure 3. Blocking probability of new customers

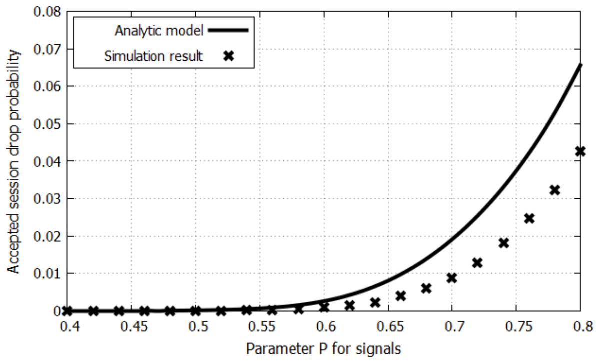

Figure 4. Blocking probability of accepted customers

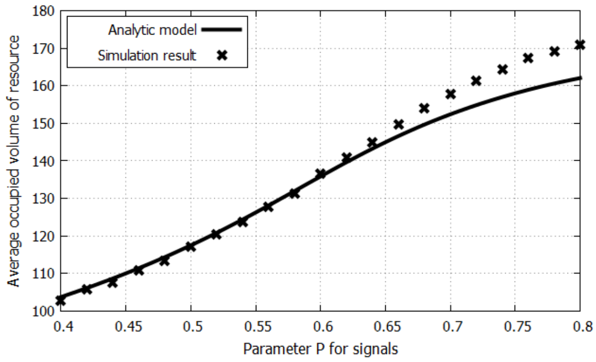

Figure 5. Average occupied volume of resource

resources and one server. This process guarantees some free volume of resource and one server for the customer, since it returns immediately with new resource requirements. But in our model for approximate analysis, the second type customers may not see any unoccupied resources on arrival. That is why the approximate approach gives the upper bound on the blocking probability of accepted customers.

V. CONCLUSION

In the paper we described an approximation approach for analysis of stationary characteristics in the queuing system with limited resources and signals. The method can be used to evaluate performance metrics in contemporary wireless network.

In our further work, we plan to implement a more accurate analysis of the analytical model with signals.

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