DETERMINATION OF SYNCHRONOUS MACHINE'S CHARACTERISTICS BASED ON THE RESULTS OF THE MATHEMATICAL MODELLING OF THE MAGNETIC FIELD

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ABSTRACT

This paper describes a methodology for determining the interdependencies between individual characteristics of synchronous machines using numerical experiments and for replacing the interdependencies with the analytical expression in a polynomial format. This approach facilitates further investigations, since it allows to replace real experiments. The proposed methodology is easy to implement with computer technologies.
The method is based on the mathematical modelling of magnetic field using partial differential equations of derivatives in relation to the magnetic vector potential, realized by the method of finite element. This method, which is described here and proven theoretically, offers how to obtain the vectoral magnetic potential as a spatial coordinate and a time function $A(x,y,t)$ as well as to determine its characteristics by mathematical modelling of stationary magnetic field.
Offered, in theory proved, and described method proposes how from a stationary magnetic field's mathematical modelling the vector magnetic potential as a spatial coordinates and as a time function, necessary characteristics are obtained.

INTRODUCTION

A certain group of the various synchronous machine's characteristics represents the connections between the armature voltage *U*, armature current *I*, excitation current I_f and power factor $\cos\varphi$. These values in stationary condition define important values for operation as consumed (engine mode) or produced (generator mode) active and reactive power, as well as the electromagnetic moment.
The characteristic can be obtained experimentally, but this is impossible at the design stage, particularly while developing a new non-traditional design when prototype production appears labour-demanding and expensive.
In present circumstances, the most promising method is mathematical modelling of the electromagnetic field with quantitative methods. Computer-based mathematical modelling techniques allow calculation of the model of the real machine with high precision. It is possible to reproduce precisely the geometrical configuration of individual elements of magnetic system as well as to reproduce the non-linear influence consideration characteristics of magnetic materials.
The magnetic field's mathematical modelling is based on the finite element method, implemented by using the multi-purpose software package QuickField.
Mathematical modelling is based on the stationary field equation considering the vector magnetic potential, which, assuming that the machine's field is plane-parallel, has only one component A (Bianchi 2005):

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu j_a , \qquad (1)$$

where μ – magnetic permeability, j_a – current density of field sources in armature winding slots and excitation winding (Fig.1.).

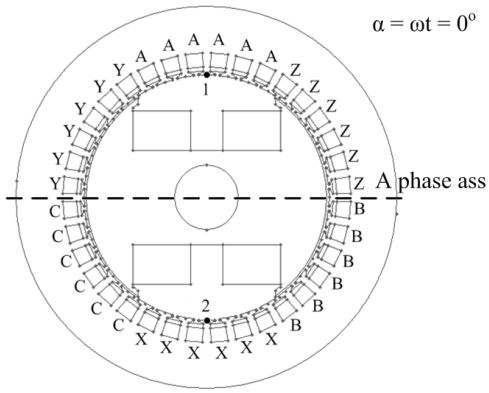

Fig. 1. The synchronous machine model ($2p = 2$).

While simulating the synchronous machine's magnetic field under load (the armature current $I_a \neq 0$ and excitation current $I_f \neq 0$), equation (1) needs to be solved *n* times, at fixed time moments $t_1, t_2, ..., t_k, ...t_n$ each time at the right side of the equation, an

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instantaneous current density values $j_a(t_1) = j_{am}\sin\omega t_1$, $j_a(t_2) = j_{am}\sin\omega t_2$, ... After solving this equation it is possible to obtain task vector potentials values. These values can be displayed in the table form as a functional connection of spatial coordinates x_i and y_i in fixed time moments t_k. In addition, it is necessary to take into account the rotor position change in relation to the armature field axis, which rotates with synchronous speed.

Determination of synchronous machine parameters and the characteristics often requires determination of resultant magnetic flux in the air gap or the one pole flux Φ_δ, which flows into the stator. The following formula is used to determine the magnetic flux: $\Phi_\delta = (A_j - A_i)l$, where A_j and A_i are the magnetic vector potential values on the stator surface at points 1 and 2 (Fig.1), l – the length of the machine in axial direction.

In this case, due to rotor's symmetry $A_1 = -A_2$, where A_1 and A_2 are the magnetic vector potential values at points 1 and 2. Considering that, magnetic flux in the air gap can be calculated as

$$\Phi_\delta = 2A_1 l. \tag{2}$$

PROCESSING METHODS OF MODELLING RESULTS AND DETERMINATION OF CHARACTERISTICS

By examining the magnetic vector potential distribution on the stator surface and by implementing the harmonic analysis, the first and the higher harmonics amplitude and phase values are obtained. After that the first harmonic e.m.f. calculation is possible using the formula (Mesņajevs and Zviedris 2010)

$$E_{\delta 1} = 4{,}44 f_1 k_w \frac{2p_1 q_1 w_{sp1}}{a_1} 2A_{1m} l, \tag{3}$$

where A_{1m} - the first harmonic amplitude value of the vector magnetic potential; f_1 - network frequency; k_w - winding factor; $\frac{2p_1 q_1 w_{sp1}}{a_1}$ - number of turns of one phase of the armature winding.

One of the research tasks is to determine the excitation winding's current I_f (or magnetization force F_f of excitation winding), which provides the given voltage U, the armature current I_a and the power factor $\cos\varphi$.

Synchronous machine armature and excitation windings are spatially differently positioned and, therefore, the winding magnetization forces produce different flux sizes in the air gap. To be able to use the above mentioned value, it is necessary to reduce armature magnetization currents or magnetization force to the excitation winding, taking into account differences of the armature field and the excitation field by the form factor (Voldek 1974).

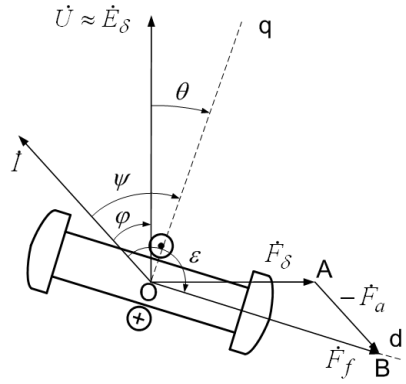

Fig.2.The vector diagram of synchronous machine.

There is a complex and non-linear connection Between E_δ and F_δ. It may be expressed only in a numerical form. When the finite element method is applied, magnetic field calculations I_f (or F_f) can only be determined by iterative procedures. To rationalize the process 3 ε values (e.g. $\varepsilon_1 = \varepsilon_{\min}$, $\varepsilon_3 = \varepsilon_{\max}$, $\varepsilon_2 = (\varepsilon_{\min} + \varepsilon_{\max})/2$, $\varepsilon_{\min}$ and $\varepsilon_{\max}$ values can be selected from the synchronous machine vector diagram in the unsaturated condition (Mesņajevs and Zviedris 2010)) and the excitation 3 current I_f values (e.g. $I_{f1} = I_{f\min}$, $I_{f3} = I_{f\max}$, $I_{f2} = (I_{f\max} + I_{f\min})/2$, $I_{f\min}$ and $I_{f\max}$ values can be selected within the range $0{,}5 \le I_f \le 2{,}5 I_{f\,N}$), there are obtained $3 \times 3 = 9$ ε and I_f combinations, which must be calculated from magnetic field. These boundaries are not defined and, if necessary, can be changed.

After the calculations of magnetic fields were made (for example using finite element method) it is necessary to determine U and φ values which are given in table 1 for each combination.

Table 1 U and φ combination table

I_f \ ε	ε_1	ε_2	ε_3
I_{f1}	U_{11} / φ_{11}	U_{12} / φ_{12}	U_{13} / φ_{13}
I_{f2}	U_{21} / φ_{21}	U_{22} / φ_{22}	U_{23} / φ_{23}
I_{f3}	U_{31} / φ_{31}	U_{32} / φ_{32}	U_{33} / φ_{33}

The table above describes the two-argument functions: $U_{i,k} = f_1(I_{fi}, \varepsilon_k)$ and $\varphi_{i,k} = f_2(I_{fi}, \varepsilon_k)$. Where using these functional connections, it is necessary to find ε and I_f values, which provide pre-set U and φ values (for example: $U = U_N$ and $\varphi = 0$). If this function is set as an analytical expression, the task reduces to the non-linear equation system solution in relation to the ε and I_f sizes (Zviedris 1999). For the approximation of the two-argument function the following polynomials can be used:

$$U = a_1 + a_2\varepsilon + a_3 I_f + a_4 \varepsilon I_f + a_5 \varepsilon^2 + a_6 I_f^2 + \ldots$$
$$\ldots + a_7 \varepsilon^2 I_f + a_8 \varepsilon I_f^2 + a_9 \varepsilon^2 I_f^2, \quad (4)$$

$$\varphi = b_1 + b_2\varepsilon + b_3 I_f + b_4 \varepsilon I_f + b_5 \varepsilon^2 + b_6 I_f^2 + \ldots$$
$$\ldots + b_7 \varepsilon^2 I_f + b_8 \varepsilon I_f^2 + b_9 \varepsilon^2 I_f^2, \quad (5)$$

where the coefficients a_1, a_2, ..., a_9, b_1, b_2, ..., b_9 determined by solving 9 polynomial equation systems inserting the appropriate $U_{i,k}, \varphi_{i,k}, I_{fi}$ and ε_k value.

For determination of these coefficients, the equation system can be solved in the matrix form:

$$ZA = U \quad (6)$$

and

$$ZB = \varphi, \quad (7)$$

where Z is a square matrix:

$$Z = \begin{bmatrix} 1 & \varepsilon_1 & I_{f1} & \varepsilon_1 I_{f1} & \varepsilon_1^2 & I_{f1}^2 & \varepsilon_1^2 I_{f1} & \varepsilon_1 I_{f1}^2 & \varepsilon_1^2 I_{f1}^2 \\ 1 & \varepsilon_2 & I_{f1} & \varepsilon_2 I_{f1} & \varepsilon_2^2 & I_{f1}^2 & \varepsilon_2^2 I_{f1} & \varepsilon_2 I_{f1}^2 & \varepsilon_2^2 I_{f1}^2 \\ 1 & \varepsilon_3 & I_{f1} & \varepsilon_3 I_{f1} & \varepsilon_3^2 & I_{f1}^2 & \varepsilon_3^2 I_{f1} & \varepsilon_3 I_{f1}^2 & \varepsilon_3^2 I_{f1}^2 \\ 1 & \varepsilon_1 & I_{f2} & \varepsilon_1 I_{f2} & \varepsilon_1^2 & I_{f2}^2 & \varepsilon_1^2 I_{f2} & \varepsilon_1 I_{f2}^2 & \varepsilon_1^2 I_{f2}^2 \\ 1 & \varepsilon_2 & I_{f2} & \varepsilon_2 I_{f2} & \varepsilon_2^2 & I_{f2}^2 & \varepsilon_2^2 I_{f2} & \varepsilon_2 I_{f2}^2 & \varepsilon_2^2 I_{f2}^2 \\ 1 & \varepsilon_3 & I_{f2} & \varepsilon_3 I_{f2} & \varepsilon_3^2 & I_{f2}^2 & \varepsilon_3^2 I_{f2} & \varepsilon_3 I_{f2}^2 & \varepsilon_3^2 I_{f2}^2 \\ 1 & \varepsilon_1 & I_{f3} & \varepsilon_1 I_{f3} & \varepsilon_1^2 & I_{f3}^2 & \varepsilon_1^2 I_{f3} & \varepsilon_1 I_{f3}^2 & \varepsilon_1^2 I_{f3}^2 \\ 1 & \varepsilon_2 & I_{f3} & \varepsilon_2 I_{f3} & \varepsilon_2^2 & I_{f3}^2 & \varepsilon_2^2 I_{f3} & \varepsilon_2 I_{f3}^2 & \varepsilon_2^2 I_{f3}^2 \\ 1 & \varepsilon_3 & I_{f3} & \varepsilon_3 I_{f3} & \varepsilon_3^2 & I_{f3}^2 & \varepsilon_3^2 I_{f3} & \varepsilon_3 I_{f3}^2 & \varepsilon_3^2 I_{f3}^2 \end{bmatrix} \quad (8)$$

but matrixes A, B, U, φ are the column matrixes

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix}, \quad (9) \qquad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \end{bmatrix}, \quad (10)$$

$$U = \begin{bmatrix} U_{11} \\ U_{12} \\ U_{13} \\ U_{21} \\ U_{22} \\ U_{23} \\ U_{31} \\ U_{32} \\ U_{33} \end{bmatrix}, \quad (11) \qquad \varphi = \begin{bmatrix} \varphi_{11} \\ \varphi_{12} \\ \varphi_{13} \\ \varphi_{21} \\ \varphi_{22} \\ \varphi_{23} \\ \varphi_{31} \\ \varphi_{32} \\ \varphi_{33} \end{bmatrix}. \quad (12)$$

When the coefficients a_i and b_i are found, they are to be placed in each of the second power degree polynomial right side, but on the left a pre-set U and φ values. Thus non-linear equations system is acquired, which must be solved with regard ε and I_f. In order to solve these systems the two-argument function minimization with an interval exclusion method can be used.

In the interval exclusion method, there is used the unimodal characteristic of function. This feature, comprising the function $f(x)$ values at two different points in the search range allows determining in which of these sub-intervals a function's minimum does not exist. Interval exclusion method's positive feature is that this method requires only the calculation of function value. It is not important if this function can be differentiated, as well as it is allowed that the function is not defined in analytical form. The only requirement is that it must be possible to calculate values of function $f(x)$ in chosen searching points.

The unimodal characteristic of the effectiveness function also can be used for two-argument functions. Variables ε and I_f of the two-argument function while there are changes in the area $\varepsilon_{\min} < \varepsilon < \varepsilon_{\max}$ and $I_{f_{\min}} < I_f < I_{f_{\max}}$, are unimodal, if this function in the given area has only one minimum, which corresponds to the argument value ε^* and I_f^*, or in this area the correlation (13) is correct for any ε and I_f values.

$$f(\varepsilon^*, I_f^*) < f(\varepsilon, I_f) \quad (13)$$

Fig. 3 presents the flowchart of the proposed two-argument function minimization algorithm.

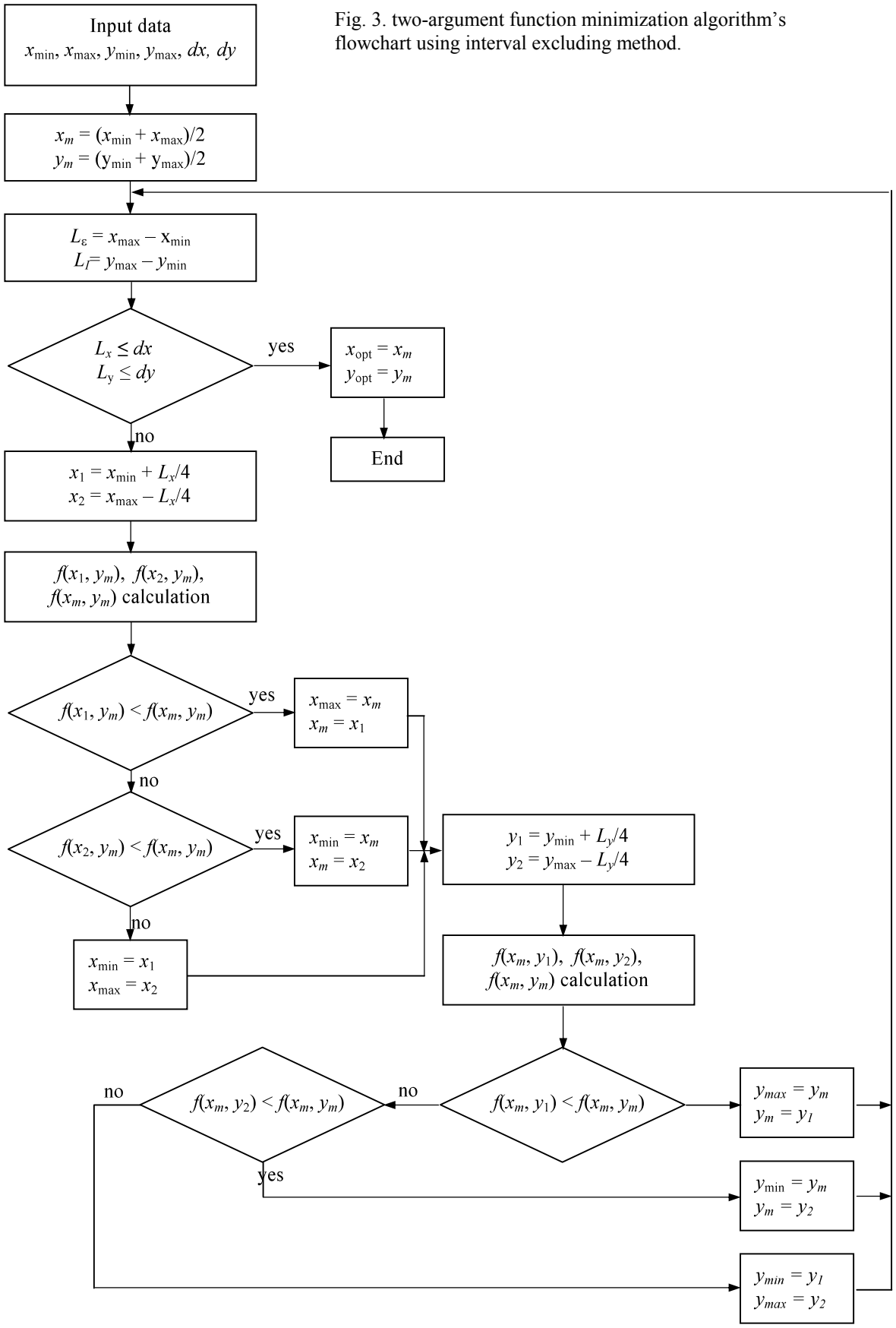

Fig. 3. two-argument function minimization algorithm's flowchart using interval excluding method.

The method described has been implemented by FORTRAN software. After the execution of the program it produces ε and I_f values which provide the pre-set U and φ values. For verification, it is desirable to make backwards calculation to make sure that the program is working correctly and the calculated coefficients provide sufficient approximation accuracy.

EXAMPLE OF CALCULATION

3 ε values ($\varepsilon_1 = 100^\circ, \varepsilon_2 = 120$, $\varepsilon_3 = 140^\circ$), and 3 excitation current I_f values ($I_{f1} = 4A$, $I_{f2} = 7A$, $I_{f3} = 10A$) were chosen. The armature current $I_a = 6{,}56A$, the voltage $U = 100V$ and the power factor $\cos\varphi = 1$ (or $\varphi = 0^\circ$) were set.

The results of the magnetic field calculation are summarized in the Table 2.

Table 2. Results of the magnetic field calculation

I_f \ ε	100°	120°	140°
4 A	96,58 V / -10,6°	76,86 V / 4,3°	52,16 V / 18,6°
7 A	125,89 V / -1,7°	116,57 V / 17,9°	108,18 V / 39,6°
10 A	142,96 V / 2,1°	137,27 V / 22,3°	131,36 V / 43,6°

Values of coefficients *a* and *b* determined using Table 2 data are given in the Table 3.

Table 3. A and B matrix coefficients

Matrix A		Matrix B	
a1	-0,97959	b1	-3,33377
a2	2,487285	b2	2,850556
a3	2,934605	b3	2,313531
a4	-3,085	b4	-2,66278
a5	-0,97719	b5	-0,71301
a6	-0,74829	b6	-0,62211
a7	1,030619	b7	0,905592
a8	0,815652	b8	0,722222
a9	-0,25738	b9	-0,23555

After the run of developed FORTRAN program, the verification calculations were made. The results are given in table 4.

Table 4 Comparison of the given and obtained values

Given values	Regime parameters	Values obtained after verification calculation
$I_{f1} = 4{,}87A$	$U = 100V$	$U = 101{,}4V$
$\varepsilon_1 = 108^\circ$	$\varphi = 0^\circ$	$\varphi = 1{,}1^\circ$

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CONCLUSIONS

As it can be seen from the table 4, the method provides sufficiently high precision.

The method is applicable for synchronous machines' (including the atypical structure) calculations.

Using mathematic modelling of the magnetic field and the developed method allow determining of characteristics of synchronous machines taking into consideration individual elements geometric shapes and saturation of magnetic system.

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ALEKSANDRS MESŅAJEVS was born in 1985 in Latvia. In 2008 he graduated Riga Technical University, obtaining the M.Sc.ing. degree. Presently he is a PhD student.

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After graduating the RPI, A.Zviedris has worked as an Assistant Professor (1968-1973), and Head of the Department of Electrical Machines and Apparatus (1973-1984). In 2001 he was elected as Associate Professor in the Department of Electrical Machines and Apparatus, where he works up to date.

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