# NEW APPROACH TO THE FORMATION OF THE ADEQUATE DIAGNOSTIC MATRIX OF THE GAS TURBINE ENGINE

Sergey M. Yunusov
Transport and Telecommunication
Institute
1 Lomonosov str., Riga LV-1019,
Latvia
E-mail: yunusov@inbox.lv

Sharif E. Guseynov
Institute of Mathematical Sciences and
Information Technologies
University of Liepaja
14 Liela str., Liepaja LV-3401, Latvia
E-mail: sh.e.guseinov@inbox.lv

## KEYWORDS

Diagnostic model, gas turbine engine, diagnostic matrix, regularization method, regularization parameter.

## ABSTRACT

This paper presents the new approach to the formation of the gas turbine engine diagnostic matrix employing A.Tikhonov's regularization method and taking into account the compressor properties shift under the condition of engine air-gas channel alteration. This method allows eliminating the certain inadequacy of the diagnostic matrices in some cases and removes the restrictions on their implementation for gas turbine engines diagnostics. The elaborated regularization algorithm of the calculation-identification matrix reversion permits to determine the diagnostic matrix persistently. The suggested method of registration of the compressor properties shift allows providing the adequacy of the engine mathematical model taking into consideration the depreciation of the engine and air-gas channel and consequently obtaining the adequate diagnostic matrix.

It is offered to employ the obtained diagnostic model in the on-board systems of the gas turbine engine control and diagnostics.

## INTRODUCTION

Gas turbine engine is a complex technical system, operation of which is determined by the whole range of the complicated physical processes, interconnected and interacting tightly. The contemporary gas turbine engines comprise a lot of components and have a complicated structure; these components interact continuously with the external environment in the process of operation, and cooperate with other sub-systems of the aircraft as well. The accurate forecast cannot be done for the further technical state of such complicated system as aircraft engine since it is impossible to present the full comprehensive description of the impact different factors can have on the engine in different situations. Accordingly, there is a necessity to take the decision on the engine state under the circumstance of indeterminacy.

The task solved by the diagnostics system in the operation process concerns receiving the data necessary for exact decision taking the further engine running as an object of diagnostics, and for possible maximal decreasing the existing indeterminacy.

Nowadays there are different methods and diagnostic models for control and diagnostics of the gas turbine engines, but it is worth mentioning that there is no universal method or model. That is the reason why the diagnostics system comprises as a rule the whole complex of models and methods employed for determining and forecasting the engine technical state.

Mathematical models as a means of forecasting the object state possess an important advantage – it is the possibility of examining the engine state under the influence of different factors under different conditions. The issue of developing the general mathematical model is identification and discovering connections between the fact of malfunction appearance and the engine parameters alteration as an object of diagnostics. The parameters under control are chosen on this purpose, then the algorithms of parameters transformation are worked out, the accuracy of measurement and the diagnostics regularity are specified, the complete association between the diagnostics results and the engine technical state are established.

One of the methods of the engine air gas channel diagnostics is the method of diagnostic matrices. The essence of the method is the following: the equations in minor deviations describing the gas turbine engine become the basis for constructing and forming the diagnostic matrix $C = A^{-1}B$, where the matrix $A$ components comprise the coefficients of the calculated parameters, and the matrix $B$ components comprise the coefficients of measured parameters.

As it was said above the task of determination of the gas turbine engine diagnostic matrix is an equivalent of the task of the stable reversion of the calculation identification matrix which as a rule occurred to be the ill-conditioned one (here is the matrix of the first type) or the matrix, the components of which fully or partly are specified approximately (here is the matrix of the first type also) or it is the underdetermined matrix since the values of certain components are not identified (here is the matrix of the second type). Consequently, for its stable reversion it is necessary to employ the special methods implementing the apparatus of the poorly conditioned problems theory (Morozov 1984, Engl et al 1985). In case when the calculation identification matrix is the matrix of the first type, the general theoretical fundamental of constructing the regulating reversed

calculation identification matrix is the following (Tikhonov et al 1986): the element/vector $0 \neq u \notin U$ is given, where $U$ is Hilbert space (in this case it is finite dimensional) and the equation

$$Az = u, \tag{1}$$

is considered, where $A : Z \to U$ is a linear operator (in this case it is finite dimensional, in other words matrix), $Z$ is Hilbert space, and $z \notin Z$ is the unknown desired element/vector. Then finding the element $z \notin Z$ which is to be continuously dependent on the right part $u$ of the equation (1) will result in determining the stable reversed operator $A^{-1}$. For fulfilling this, instead of the operator equation (1) the optimization problem

$$\|Az - u\|_U \to \min_{z \notin Z} \tag{2}$$

is considered $\{R\}$ marks the class of these positive definite self-adjoint operators $Q$ that the quadric form $\|Az\|_U^2 + (Qz, z)_Z$ is not less than $\beta^2 \cdot \|z\|_Z^2$, in other words $\|Az\|_U^2 + (qz, z)_Z \geq \beta^2 \cdot \|z\|_Z^2$, where $\beta = \beta(A, Q) > 0$ is a constant, not dependent on $z \notin D(Q)$. The composed function is introduced:

$M[Q; z, u] \stackrel{def}{\equiv} \|Az - u\|_U^2 + (Qz, z)_Z$, $z \notin D(Q)$. Then the element $z_{ex} \notin D(Q)$, minimizing the composed function $M[Q, z, u]$, satisfies the equation $(Q + A^* A) z = A^* u$, having the unique solution $z = R(Q) u$, where $R(Q) \stackrel{def}{\equiv} (Q + A^* A)^{-1} A^*$. Then the element $u$ is given approximately, in other words it is assumed that $\tilde{u} = u + \delta$, where $\delta$ is a certain random process with the values in $U$, for which the probabilistic average is zero, in other words $E\{\delta\} = 0$. It is marked

$$\Delta^2[Q; z(u); \delta] \stackrel{def}{\equiv} E\left\{\|R(Q)\tilde{z} - z(u)\|_z^2\right\}$$

and

$$\tilde{\Delta}^2[Q; \{z(u)\}; \{\delta\}] \stackrel{def}{\equiv} \sup_{\substack{z(u) \in \{z(u)\}, \\ \delta \in \{\delta\}}} \Delta^2[Q; z(u); \delta],$$

where $\{z(u)\}$ is the class of the admissible solutions of the task (2), and $\{\delta\}$ is the class of the admissible perturbations of the optimization task (2). The optimal regularization is the operator

$$Q^{opt} = \arg \inf_{Q \in \{Q\}} \tilde{\Delta}[Q; \{z(u)\}; \{\delta\}],$$

in other words $Q^{opt} \in \{Q\}$ is determined as the task solution

$$\tilde{\Delta}[Q; \{z(u)\}; \{\delta\}] \to \inf_{Q \in \{Q\}}. \tag{3}$$

If the solution of the extreme task (3) exists, in this case the element $z^{opt} \in R^{opt}(Q)\tilde{u}$ is called as $(Q; \{z(u)\}; \{\delta\})$ – an optimal regularized solution of task (2). In this case the value $\tilde{\Delta}^{opt}[Q^{opt}; \{z(u)\}; \{\delta\}] \stackrel{def}{\equiv} \inf_{Q \in \{Q\}} \tilde{\Delta}[Q; \{z(u)\}; \{\delta\}]$ is error $(Q; \{z(u)\}; \{\delta\})$ – the optimal regularized solution.

In case when $\{Q\} = \{Q : Q = \alpha \cdot I\}$, where $I$ is a unity operator, and $\alpha > 0$ there is a certain parameter (in general, it is unknown), called the parameter of regularization, and the extreme task (3) is reduced to the determination of the optimal value of the regularized parameter $\alpha = \alpha^{opt}$. Then it is assumed that $z(u)$ is the solution for task (2) with the minimal admissible norm, in other words it is assumed that

1.  $\{Q\} = \{Q : Qe_i = \alpha_i \cdot e_i, i \in \}$, where $\{e_i\}_{i \in}$ is orthonormalized system of own values of the self-adjoint operator $A^* A$: $A^* Ae_i = \lambda_i \cdot e_i, i = 1, 2, ...;$ $\lambda_i > 0, i \in ; \{\lambda_i\} \downarrow$.

2.  $\delta \in \{\delta\}_\gamma$, where
    $$\{\delta\}_\gamma \stackrel{def}{\equiv} \left\{\delta_i : E\{\delta_i^2\} = \gamma_i^2, 0 < \gamma \geq \sup_{i \in} \delta_i^2\right\}.$$

Then $(Q; \{z(u)\}; \{\delta\})$ is an optimal regularized solution $z^{opt}$ is expressed by the formula

$$z^{opt} = \sum_{i=1}^{\infty} \frac{u_i^2 \cdot (u_i + \delta_i)}{\sqrt{\lambda_i} \cdot (u_i^2 + \gamma_i^2)} \cdot e_i, \tag{4}$$

where

$$\left.\begin{aligned} &\left(\tilde{\Delta}^{opt}[Q^{opt}; \{z(u)\}; \{\delta\}]\right)^2 = \sum_{i=1}^{\infty} \frac{\gamma_i^2 \cdot u_i^2}{\lambda_i \cdot (u_i^2 + \gamma_i^2)}, \\ &\alpha_i^{opt} = \frac{\lambda_i \cdot \gamma_i^2}{u_i^2}, i \in . \end{aligned}\right\} \tag{5}$$

From (4), (5) it is possible to receive the formula for $(Q; \{z(u)\}; \{\delta\}_\gamma)$ - the optimal regularized solution $z_\gamma^{opt}$:

$$z_\gamma^{opt} = \sum_{i=1}^{\infty} \frac{u_i^2 \cdot (u_i + \delta_i)}{\sqrt{\lambda_i} \cdot (u_i^2 + \gamma^2)} \cdot e_i, \tag{6}$$

where

$$\left.\begin{aligned} &\left(\tilde{\Delta}^{opt}[Q^{opt}; \{z(u)\}; \{\delta\}_\gamma]\right)^2 = \\ &= \gamma^2 \cdot \sum_{i=1}^{\infty} \frac{u_i^2}{\lambda_i \cdot (u_i^2 + \gamma^2)} \xrightarrow{\gamma \to 0} 0; \\ &\alpha_i^{opt} = \gamma^2 \cdot \frac{\lambda_i}{u_i^2}, i \in . \end{aligned}\right\} \tag{7}$$

It is obvious, that in this case – in case of the finite dimensionality of the operator $A : Z \to U$ from the

original equation (1), or when $A = \{a_{ij}\}_{i,j=\overline{1,N}}$, – the above demonstrated formulae (4), (5) and (6), (7) stand in force, only if the lines in these formulae are exchanged by the corresponding sums consisting of $N$ summands. For instance, for this case $\left(Q; \{z(u)\}; \{\delta\}_\gamma\right)$ – is the optimal regularized solution $z_\gamma^{opt}$ of matrix equation (1) will be the regularized solution of the equation $\left(A^*A + \alpha \cdot I\right)z = A^*\left(u + \delta\right)$ when $\alpha = \alpha^{opt} = \dfrac{\gamma^2}{c^2}$, and the error $\left(\tilde{\Delta}^{opt}\left[Q^{opt}; \{z(u)\}; \{\delta\}_\gamma\right]\right)^2$ will be the following (Guseynov at al 2003):

$$\left(\tilde{\Delta}^{opt}\left[Q^{opt}; \{z(u)\}; \{\delta\}_\gamma\right]\right)^2 = \gamma^2 \cdot c^2 \cdot \sum_{i=1}^{N} \frac{1}{\gamma^2 + c^2 \cdot \lambda_i^2},$$

where $\{z(u)\}_N \overset{def}{\equiv} \{z(u): z(u) \in Z, u_i^2 \le c^2, i = \overline{1,N}, c = const.\}$

Imposing these or those restrictions on the coefficients rate of decay $u_i, i = \overline{1,N}$ and the values $\gamma_i, i = \overline{1,N}$, it is possible to receive the exact values for the error $\tilde{\Delta}^{opt}\left[Q^{opt}; \{z(u)\}; \{\delta\}\right]$. The particularization of such type is the essence of the following two parts of the work under consideration.

## FORMATION OF THE DIAGNOSTIC MATRIX FOR CONTROLLING THE STATE OF THE GAS TURBINE ENGINE AIR GAS CHANNEL

### Developing the Adequate Mathematical Model of the Engine for Diagnostic Matrix Formation

Since Urban (Urban 1974, 1982) introduced his gas path analysis in the 1970's, a substantial number of papers have been published in this area. These papers have proposed a wide variety of algorithms, employing linear (Doel 1994, Xia et al 2009) and non-linear methods (Stamatis et al 1990), genetic algorithms (Zedda et al 1999, 2002), neural networks (Denny 1965) and fuzzy expert systems (Fuster et al 1997, Siu et al 1997). Comparisons of these methods were extensively reviewed in (Li 2002, Kamboukos et al 2005). Currently, gas path analysis is used in gas turbine analysis both widely and commercially. All the peculiarities and properties of the diagnostic matrices useful for practice are the results of the analysis of the mathematical model of the processes taking place in the air gas channel of the gas turbine engines. The equations describing the processes in the components of the gas turbine engine are complicated, some of them are given graphically (for example, the compressor features). The operational process of the gas turbine engine represents the aggregate of the whole range of closely interconnected complex processes, and changing even single of the gas turbine engine parameters – sectional area, loss coefficient, coefficient of efficiency etc. – results in changing almost all other parameters of the flow (pressure, temperature, velocities) and further in the changes of the engine properties – thrust, power, fuel consumption. For simplifying the analysis of

dependence between the increments in the interconnected equation parameters the approximate mathematical method is implemented – the method of minor deviations, allowing obtaining the linearization of the process original equations. As an example there is the way of obtaining the equations in minor deviations from the process principal equations. The work, spent for compressing 1 kilo of air in the compressor is expressed via the equation:

$$L_c = \frac{k}{k-1} RT_{t1}\left(\pi_c^{0.286} - 1\right)\frac{1}{\eta_c},$$

where
$T_{t1}$ – is the temperature of the stagnated flow before the compressor,
$\pi_c$ – is the degree of increasing the full pressure in the compressor,
$\eta_c$ – is the compressor adiabatic efficiency on the stagnated flow parameters.
After taking the logarithms of the right and the left parts of the equation

$$ln\, L_c = ln\left(\frac{k}{k-1}R\right) + ln\, T_{t1} + ln\left(\pi_c^{0.286} - 1\right) - ln\, \eta_c$$

and then differentiating the received equation and taking into consideration that

$$d\left(ln\, x\right) = \frac{dx}{x},$$

it is possible to discover

$$\frac{dL_c}{L_c} = \frac{dT_{t1}}{T_{t1}} + \frac{0.286\pi_c^{0.286}}{\pi_c^{0.286}}\frac{d\pi_c}{\pi_c} - \frac{d\eta_c}{\eta_c}$$

Though, the connection between the relative changes is found, and the following notions are introduced:

$$\frac{dL_c}{L_c} \approx \frac{\Delta L_c}{L_c} = \delta L_c$$

$$\frac{dT_{t1}}{T_{t1}} \approx \frac{\Delta T_{t1}}{T_{t1}} = \delta T_B$$

$$\frac{d\eta_c}{\eta_c} \approx \frac{\Delta \eta_c}{\eta_c} = \delta\eta_c$$

$$\frac{d\pi_c}{\pi_c} \approx \frac{\Delta \pi_c}{\pi_c} = \delta\pi_c$$

then

$$\delta L_c = \delta T_{t1} + K_1\delta\pi_c - \delta\eta_c$$

This is the final equation of the process of compressing in minor deviations. The coefficients of $\delta T_{t1}, \delta\pi_c, \delta\eta_c$ in this equation are the coefficients of influence on $L_c$. It is obvious, that in $\delta T_{t1}$ the coefficient is 1, and in $\delta\eta_c$ it is -1, and in $\delta\pi_c$ it is $K_1$

$$K_1 = \frac{0.286\pi_c^{0.286}}{\pi_c^{0.286} - 1}$$

The equation in minor deviations should be understood in the following way: if the air temperature $T_{t1}$ increases by 1%, the operation of compressing, ceteris paribus, $(\pi_c = const, \eta_c = const, \delta\pi_c = \delta\eta_c = 0)$ increases by 1%. Similarly to this, in case of increasing $\eta_c$ by 1% the operation of compressing decreases by 1%, and if $\pi_c$ increases by 1% the operation of compressing increases by $K_1\%$. So the equation allows the determination of changing the operation of $L_c$ with simultaneous alteration of $T_{t1}, \pi_c, \eta_c$, as well as finding the connection between the changes of $T_{t1}, \pi_c$ and $\eta_c$ under the determined change in the compression operation, in other words finding solution for the reversed task. As an instance there is a mathematical model below. This model was obtained with the implementation of the method of minor deviations for the turbo propeller engine TB7-117C, presented in the following form

1. the equation of parity of the compressor and the turbine operations (absence of bypassing and air bleeding):

$$K_1\delta\pi_c - \delta\eta_c = \delta T_{t4} + K_3\delta\pi_T + \delta\eta_T$$

2. the correlation of pressures in channel:

$$\delta\pi_{out} = \delta\pi_c - \delta\pi_T + \delta\sigma_{in} + \delta\sigma_{ch} + \delta\sigma_{out}$$

3. the equation of compression process in the compressor:

$$\delta T_{t2} = K_1 \cdot K_2 \cdot \delta\pi_c - K_2 \cdot \delta\eta_c$$

4. the equation of the extension process in turbine:

$$\delta T_{t4} = \delta T_{t3} - K_3 \cdot K_4 \cdot \delta\pi_T - K_4 \cdot \delta\eta_T$$

5. the equation of the continuity between the compressor inlet and the throat section of the turbine nozzle cascade:

$$\delta G_{air} = \delta\sigma_{in} - \delta\pi_c + \delta\sigma_{ch} + \delta F_T - 0.5\sigma T_{t3}$$

6. the equation of the continuity between the turbine nozzle cascade and the jet nozzle:

$$\delta\pi_T - \delta\sigma_{out} + \delta F_T = \delta F_{out} + K_6\delta\pi_{out}$$

7. the equations describing the compressor properties:

$$\delta G_{air} = K_{10}\delta\pi_c + \delta\sigma_{in}$$
$$\delta\eta_c = K_{11}\delta\pi_c + \delta\overline{\eta}_c$$

8. the equation of the heat input:

$$\delta G_f = \delta G_{air} + K_5\delta T_{t3} - (K_5 - 1)\delta T_c - \delta\xi_{ch}$$

9. the equation of the jet thrust:

$$\delta R = K_9\delta F_{out} + K_7 \cdot K_8 \cdot K_9\delta\pi_{pc} - (K_9 - 1)\delta G_{air}$$

Nevertheless, in the obtained linear mathematical model the alteration of the engine air gas channel state is not taken into consideration, and this fact does not allow further forming the adequate diagnostic matrix. For taking into account the changes of the engine air gas channel it is suggested using the idea of introduction of the specific components into the compressor equations.
It is assumed here that under the condition of changing the state of the compressor air gas channel its head-capacity characteristics shift in the equidistant way by value $\Delta G_{air0-1}$ horizontally and $\Delta\pi_{c0-1}$ vertically (Fig.1.)
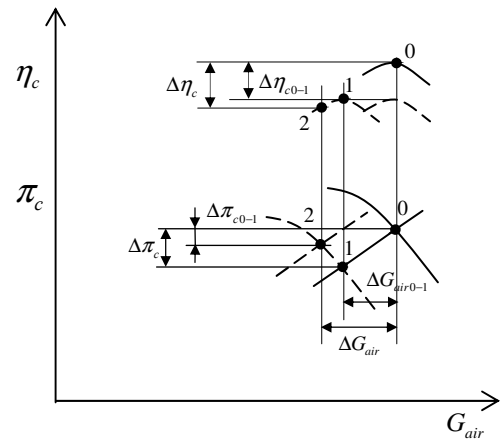


Figures 1: Diagram of Property Shifting

Besides the property shift there is also the shift of the operational modes line and the operational point on it (1-2) in the way that the cumulative change of the air consumption and the degree of pressure are correspondingly $\Delta G_{air}$ and $\Delta\pi_c$. Simultaneously there is the change in the efficiency by value $\Delta\eta_{c0-1}$, and taking into account the operational point shift - by $\Delta\eta_c$. Employing the approach (Chercez 1975), the full relative increments of the parameters in this case are determined as the sum of relative partial increments in the first (0-1) and the second (1-2) transitions:

$$\delta G_{air} = \delta G_{air0-1} + \delta G_{air1-2}$$
$$\delta\eta_c = \delta\eta_{c0-1} + \delta\eta_{c1-2}$$
$$\delta\pi_c = \delta\pi_{0-1} + \delta\pi_{c1-2}$$

Taking into consideration that

$$\delta G_{air1-2} = K_{10}\delta\pi_{c1-2};$$
$$\delta\eta_{c1-2} = K_{11}\delta\pi_{c1-2}$$
$$\delta\pi_{c1-2} = \delta\pi_c - \delta\pi_{c0-1}$$

the following correlations are received:

$$\delta G_{air} = \delta G_{air0-1} + K_{10}\left(\delta\pi_c - \delta\pi_{c0-1}\right)$$
$$\delta\eta_c = \delta\eta_{c0-1} + K_{11}\left(\delta\pi_c - \delta\pi_{c0-1}\right)$$

The parameters relative increments along the line (0-1) are marked as $\delta\overline{G}_{air}$; $\delta\overline{\eta}_c$; $\delta\overline{\pi}_c$, then

$$\delta G_{air} = \delta\overline{G}_{air} + K_{10}\left(\delta\pi_c - \delta\overline{\pi}_c\right)$$
$$\delta\eta_c = \delta\overline{\eta}_c + K_{11}\left(\delta\pi_c - \delta\overline{\pi}_c\right)$$

Is expressed $\delta\overline{G}_{air} = \overline{K}_{10}\delta\overline{\pi}_c$, where

$$\overline{K}_{10} = \frac{G_{air1} - G_{air0}}{G_{air0}}\frac{\pi_{c0}}{\pi_{c1} - \pi_{c0}} = \frac{\Delta\overline{G}_{air}}{G_{air}}\frac{\pi_{c0}}{\Delta\overline{\pi}_c}$$

This is the coefficient of the connection of alteration of the given air consumption and the degree of the compressor pressure increase along the line of shifting the head-capacity curve (it is assumed, that this shift takes place along the operational modes line). Accordingly there is the additional parameter $\Delta\overline{G}_{air}$ in the equations, and this parameter characterizes the shift of the head-capacity curves of the compressor properties. The set of equations describing the compressor properties taking into consideration the malfunctions of the compressor and correspondingly the shift of its properties is the following:

$$\delta G_{air} = K_{10}\delta\pi_c + \left(1 - \frac{K_{10}}{\overline{K}_{10}}\right)\delta\overline{G}_{air} + \delta\sigma_{in}$$
$$\delta\eta_c = K_{11}\delta\pi_c + \delta\overline{\eta}_c - \frac{K_{11}}{\overline{K}_{10}}\delta\overline{G}_{air}$$

Accordingly the original equations describing the compressor properties, in the original mathematical model are exchanged by the newly obtained ones. The obtained mathematical model will be implemented for the diagnostic matrix formation.

## General Approach to the Diagnostic Matrix Formation

The diagnostic matrix formation is demonstrated by setting the diagnostics of the air gas channel of the engine TB7-117C, for which the mathematical model was developed. It is worth mentioning that the depth of the engine air gas channel diagnostics is connected with the number of the registered fluid dynamics parameters. In this case the following parameters are supposed to be the registered (measured) ones:
1. full temperature and air pressure at the engine inlet;
2. speed of the compressor rotor;
3. full gas temperature behind the turbine;
4. full air pressure behind the compressor;
5. fuel consumption per hour;
6. speed of the free power turbine rotor.

For obtaining the diagnostic matrix the measured parameters are taken into the right part of the equation and the left part comprises the others. The necessary number of the measured parameters is determined as a difference between the number of variables and the number of equations. The number of the measured parameters needed for the diagnosis is determined by the number of independent criteria of calculation, the deviations of which characterize the state of the engine junctions. The diagnostic matrix is formed for the mode of maximum operation duration (under the condition of power N=2720 kW). The left parts of the equations are used for matrix A creation, and the right parts – for matrix B. Substituting them with the numerical values of the coefficients, calculated for this operation mode, and solving the set of equations: $C = A^{-1}B$.

The engine diagnostic matrix is obtained. Formation is described in details in the work (Labendik and Kuznetsov 1992) The first four lines of the diagnostic matrix present the information on the gas generator state. The other lines have the referential features. For examining the diagnostic matrix it is possible to simulate the gas generator malfunctions, giving any deviation of the junctions state parameters $\left(\delta\overline{G}_{air}, \delta\overline{\eta}_c, \delta F_T, \delta\eta_T\right)$.

Nevertheless, the practice has shown that the diagnostic matrix development is not always possible, that is why certain papers (Novikov et al 2007) point the restrictions of their employment. It is connected with the fact that obtaining the reversed matrix $A^{-1}$ is impossible, and consequently it is impossible to obtain the diagnostic matrix as well. Next part of the paper suggests the regularizing algorithm of obtaining the stable calculation identification matrix on the basis of Tikhonov's method.

## The Regularizing Algorithm of the Steady Inversion of the Calculation Identification Matrix

The algorithm, enables finding the inverse calculation-identification matrix, is based on the regularization method idea by academician A.Tikhonov. The put-forward algorithm comprises the following stages.

Stage 0. The calculation identification matrix is found as a result of gas-dynamic calculation, the order of this matrix is $m\times n$, where $m = n$, or $m > n$;

Stage1. First, the arbitrary vector $u$ of order $m\times 1$ is found. Then the set of simultaneous linear algebraic equations is done.

$$Az = u, \tag{8}$$

where $z$ a task is is required solution vector (8);

Stage 2. In system (8) the matrix $A$ and the right part of $u$ are disturbed:

$$\left\|A^h - A\right\| \le h, \ \left\|u_\delta - u\right\| \le \delta,$$

where $h$ and $\delta$ are supposed to be invariably defined.

As a result, instead of (8) there is the following disturbed set of simultaneous linear algebraic equations:

$$A^h\tilde{z} = u_\delta, \tag{9}$$

where $\tilde{z}$ is the required approximate set (9) solution, dependent on the inaccuracies $\{h; \delta\}$;

Stage 3. The set (9) on the left is multiplied by the transposed matrix $\left(A^h\right)^T$:

$$\left(A^h\right)^T A^h \tilde{z} = \left(A^h\right)^T u_\delta.$$

Here indicating $\tilde{A}^h = \left(A^h\right)^T A^h$; $\tilde{u}_\delta = \left(A^h\right)^T u_\delta$, the following is found:

$$\tilde{A}^h \tilde{z} = \tilde{u}_\delta; \qquad (10)$$

Stage 4. The matrix $B^\alpha = \left\{\beta_{i,j}^\alpha\right\}_{i,j=\overline{1,m}}$ is built up. The components of this matrix are detected according to the formulae (Guseynov 2003, 2004)

$$\beta_{i,j} = \sum_{k=1}^m a_{i,k} \cdot a_{j,k}, \qquad \forall j, j = \overline{1,m}.$$

The value $\alpha$ is added to the matrix components on the principal diagonal, it is the parameter of regularization.

$$B^\alpha = \begin{pmatrix} \alpha + \beta_{11} & \beta_{21} & \dots & \beta_{n1} \\ \beta_{12} & \alpha + \beta_{22} & \dots & \beta_{n2} \\ \dots & \dots & \dots & \dots \\ \beta_{1m} & \beta_{2m} & \dots & \beta_{nm} \end{pmatrix};$$

Stage 5. The regularizing parameter (matrix) $R^\alpha = \left(B^\alpha\right)^{-1} \cdot \tilde{A}^k$ is detected (Guseynov 2007);

Stage 6. The approximate solution for the set of simultaneous linear algebraic equations is found.

$$\tilde{z} = R^\alpha u_\delta$$

Having the invariable value of the rate $\delta$ and changing the value of $\alpha$, the optimal value is detected.

**Application of the Worked-out Algorithm for Finding the Steady Information-Diagnostic Matrix**

The diagnostic matrix of TB7-117C engine was obtained with this worked-out algorithm, and the number of measured parameters is four (tab.1). It is important, the matrix under consideration is the matrix of the definite operational and efficient engine.
The achieved diagnostic matrix (DM) allows analyzing the diagnostic properties, which are response for the fluid dynamic parameters' minor deviations. The diagnostic matrices allow locating mostly the charging set failure. At the engine operation process the gas generator failures develop as a rather specific, inherent for this trouble, set of deviations of measured parameters $\left(\delta G_f, \delta T_{t4}^*, \delta T_{t2}^*, \delta \pi_c^*\right)$.

Table 1: Engine Diagnostic Matrix

| | $\delta n_{Tc}$ | $\delta T_{Tc}$ | $\delta G_f$ | $\delta \pi_c$ |
|---|---|---|---|---|
| $\delta \bar{G}_{air}$ | -2.48 | -0.702 | 0 | 0.053 |
| $\delta \bar{\eta}_C$ | 0.189 | 6.736 | -8.803 | 0.482 |
| $\delta F_T$ | 0 | -1.397 | 1.287 | -1 |
| $\delta \eta_T$ | 0 | -5.667 | 6.229 | -0.551 |
| $\delta \eta_c$ | 0 | 6.809 | -8.803 | 0.553 |
| $\delta G_{air}$ | 0 | -0.757 | 0 | 0 |
| $\delta \pi_{Tc}$ | 0 | 0.257 | 0 | 1 |
| $\delta \pi_{Tf}$ | 0 | -0.257 | 0 | 0 |
| $\delta T_{t4}$ | 0 | -1.284 | 2.574 | 0 |
| $\delta T_{t2}$ | 0 | -3.871 | 5.005 | 0 |

But even the experienced engineer, an expert, cannot estimate the whole variety of these parameters deviations. The diagnostic matrix allows locating the problem in the engine air-gas channel by means of defining trouble partial criteria (for compressor they are $\delta \bar{\eta}^*$ and $\delta \bar{G}_{air}$ and for turbine they are $\delta \eta_T^*$ and $\delta F_T$. Suppose at the process of engine operation the following set of parameter deviation is received: $\delta n_{Tc} = -0.074$, $\delta T_{t4} = 1.659$, $\delta G_f = 1.472, \delta \pi_c^* = 0.426$, where $\delta n_{Tc}$ is the engine speed deviation, $\delta T_{t4}^*$ is the deviation of gas temperature behind the turbine, $\delta G_f$ is the fuel consumption deviation, $\delta \pi_c^*$ is the engine pressure ratio deviation. Multiplying the coefficients of the efficient diagnostic matrix by the received deviations of the measured parameters, and summing these products up, we obtain the deviations $\delta \bar{\eta}_c^*$ and $\delta \bar{G}_{air}$ in the engine compressor, and $\delta F_T$, $\delta \eta_{Tc}^*$ in the engine turbine.

$$\delta \bar{G}_{air} = (-2.48) \cdot (-0.074) + (-0.702) \cdot (1.659) + \\ + 0 \cdot (1.472) + (0.0503) \cdot (-0.426) = -1.00368$$

$$\delta \bar{\eta}_c^* = (0.189) \cdot (-0.074) + (6.736) \cdot (-0.702) + (-8.803) \cdot \\ \cdot (1.472) + (0.482) \cdot (-0.426) = -2.00231$$

$$\delta F_T = 0 \cdot (-0.074) + (-1.397) \cdot (1.659) + (1.287) \cdot \\ \cdot (1.472) + (-1) \cdot (-0.426) = 0.00284$$

$$\delta \eta_{Tc} = 0 \cdot (-0.074) + (6.736) \cdot (1.659) + (6.229) \cdot \\ \cdot (1.472) + (-0.553) \cdot (-0.426) = -0.003113$$

With this set of values the diagnostic matrix points the trouble in the compressor, as we see the compressor efficiency loss by 2% and air consumption rate by 1%, and the turbine is practically operational at this moment, as there is practically no any change in the passage area of the set of nozzles and no change in the turbine efficiency.

The values for every pair of criteria for every block allow introducing them as the defect field of vector, and then to estimate the development trends of these defects from flight to flight, and taking into consideration the access scope to anticipate the time of these defects dangerous development and engine malfunction. Certainly, the measured parameters deviations sets for every flight are average samples, obtained after flight information processing and referencing it to the definite engine work modes (typical operation, cruise rating, etc.).

## CONCLUSIONS

The paper under consideration presents the approach to the formation of the adequate diagnostic matrix of the gas turbine engine. For formation of the adequate mathematical model it is offered to introduce the additional components, capable of accounting the depreciation of the engine air gas channel, into the compressor equations. For determining the diagnostic matrix $C$ of the gas turbine engine the regularizing algorithm is suggested. The idea of Tikhonoff's regularizing method is the basis of the suggested algorithm. This algorithm for determination of the diagnostic matrix is universal, and this fact allows implementing it (with minimal modifications) for the tasks of determination of the diagnostic matrices for different gas turbine engines of complicated schemes. Accordingly the principal reasons restricting the diagnostic matrices implementation are eliminated. The diagnostic matrices can be introduced directly into the on-board systems of control and troubleshooting of the gas turbine engine as well as implemented for training the neural network. The neural network trained on the diagnostic matrix, permits to define the technical state of the engine under the condition of incompleteness or absence of the data on certain engine parameters. These tasks are assumed to be the identification tasks with the uncertainty.

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## REFERENCES

Chercez A. 1975. "Engineering Calculations of gas turbine engines by small deviations." Moscow, Mashinostrojenije.

Denny G. 1965. "F-16 Jet Engine Trending and Diagnostics with Neural Networks." *Applications of Neural Networks* 4, 419-422.

Doel D.L. 1994. "TEMPER – a gas-path analysis tool for commercial jet engines." *Journal of Engineering for Gas Turbines and Power* 116 (1), 82-89.

Engl H.W. and Neubauer A. 1985. "Optimal discrepancy principles for the Tikhonov regularization of integral equations of the first kind." *Constructive Methods for the Practical Treatment of Integral Equations*. (G.Hämmerlin and K.-H.Hoffmann, eds.), Birkhäuser, Basel, Boston, Stuttgart, 120-141.

Fuster P., LigezaA. and Aguilar M.J. 1997. "Adductive diagnostic procedure based on an AND/OR/NOT graph for expected behaviour: Application to a gas turbine." *10th International Congress and Exhibition on Condition Monitoring and Diagnostic Engineering Management*, Helsinki, Finland.

Guseynov Sh.E. 2003. "One simple method for solution of some class inverse problems." *Journal of Computer Modelling and New Technologies* 7 (1), 55-58.

Guseynov Sh.E. 2004. "Conservative averaging method for solutions of inverse problems of mathematical physics." *Springer Book of Progress in Industrial Mathematics*, 241-246.

Guseynov Sh.E. 2007. "On new analytical approach to some classes of mathematical physics inverse problems." *Journal of Applied Mathematics and Mechanics* 7 (2), 634-645.

Guseynov Sh.E. and Volodko I. 2003. "Convergence order of one regularization method." *Journal of Mathematical Modelling and Analysis* 8 (1), 25-32.

Kamboukos P. and Mathioudakis K. 2005. "Comparison of linear and nonlinear gas turbine performance diagnostics." *Journal of Engineering for Gas Turbines and Power* 127 (1), 49-56.

Labendik V. and Kuznetsov N. 1992. "On the development of adequate mathematical model of linear GTE for control of its flow part with diagnostic matrices." Riga, RAU. 14.

Li Y.G. 2002. "Performance-analysis-based gas turbine diagnostics: A review." *Proceedings of the Institution of Mechanical Engineers*, Part A: Journal of Power and Energy 216 (5), 363-377.

Morozov V.A 1984. "Methods of Solving Incorrectly Posed Problems." *Springer-Verlag*, New York, Berlin Heidelberg.

Novikov A.S., Paikin A.G. and Sirotin N.N. 2007 "Control and Diagnostics of the Gas-Turbine Engine Performance." Moscow: Nauka, 469.

Siu C., Shen Q. and Milne R. 1997. "TMDOCTOR: A fuzzy rule-and case-based expert system for turbomachinery diagnosis." *Proceedings of IFAC Fault Detection, Supervision and Safety for Technical Processes*, Kingston Upon Hull, UK.

Stamatis A., Mathioudakis K. and Smith M. 1990. "Gas turbine component fault identification by means of adaptive performance modeling." *Proceedings of ASME Turbo Expo*, Brussels, Belgium.

Tikhonov A.N. and Arsenin V.Ya. 1986. "Solutions ill-posed problems." Moscow: Nauka, 285.

Urban L.A. 1974. "Parameter selection for multiple fault diagnostics of gas turbine engines." *AGARD Conference Proceedings*, Liege, Belgium.

Urban L.A. 1982. "Gas path analysis of commercial aircraft engines." *Proceedings of the 11th Symposium on Aircraft Integrated Data Systems*, Cologne, West Germany.

Xia D., Wang Y. and Weng Sh. 2009. "A new method to evaluate the influence coefficient matrix for gas path analysis." *Springer Journal of Mechanical Science and Technology* 23, 667-676.

Zedda M. and Singh R. 1999. "Gas-turbine engine and sensor diagnostics." *XIV International symposium on air-breathing engines*, Florence, Italy, 255-261.

Zedda M. and Singh R. 2002. "Gas turbine engine and sensor fault diagnosis using optimization techniques." *Journal of Propulsion and Power* 18 (5), 1019-1025.

## AUTHORS' BIOGRAPHIES

**SERGEY M. YUNUSOV** was born in 1956, in the Ukraine. In 1985 he graduated with honours from the Military Aviation Engineering Academy after professor N. Zhukovsky in Moscow. Sergey Yunusov has occupied different positions and titles in the sphere of aircraft maintenance for nearly 20 years. Since 2000, he has been going in for pedagogical work and possesses Master's Degree in the field of Computing. His scientific interests are connected with developing and projecting data bases, expertise systems and applying intellectual means for aviation Gas-Turbine Engines Diagnostics. Sergey is a doctoral student of the Transport and Telecomunication Institute. His e-mail address is yunusov@inbox.lv and advance information can be found at www.tsi.lv (office-page).

**SHARIF E. GUSEYNOV** was born in 1967, in Azerbaijan. In 1997 he graduated from the Lomonosov Moscow State University in Moscow, Russia. His research works are to solving the Inverse and Ill-posed Problems; Partial Differential Equations; Equations of Mathematical Physics; Mathematical Modelling; Integral and Operator Equations, Nonlinear Physics; Functional Analysis and Applications; Computational Methods; Mathematical Economics; Optimal Control and Optimization Methods; Game Theory and Operations Research. Dr.Sc.Math. Sharif Guseynov published for more than 200 articles in various scientific journals; 1 monograph and 3 textbooks. His e-mail address is sh.e.guseinov@inbox.lv. More information about Prof. Sh.E.Guseynov can be found at www.guseynov.lv (home-page) and www.imsit.liepu.lv (office-page).